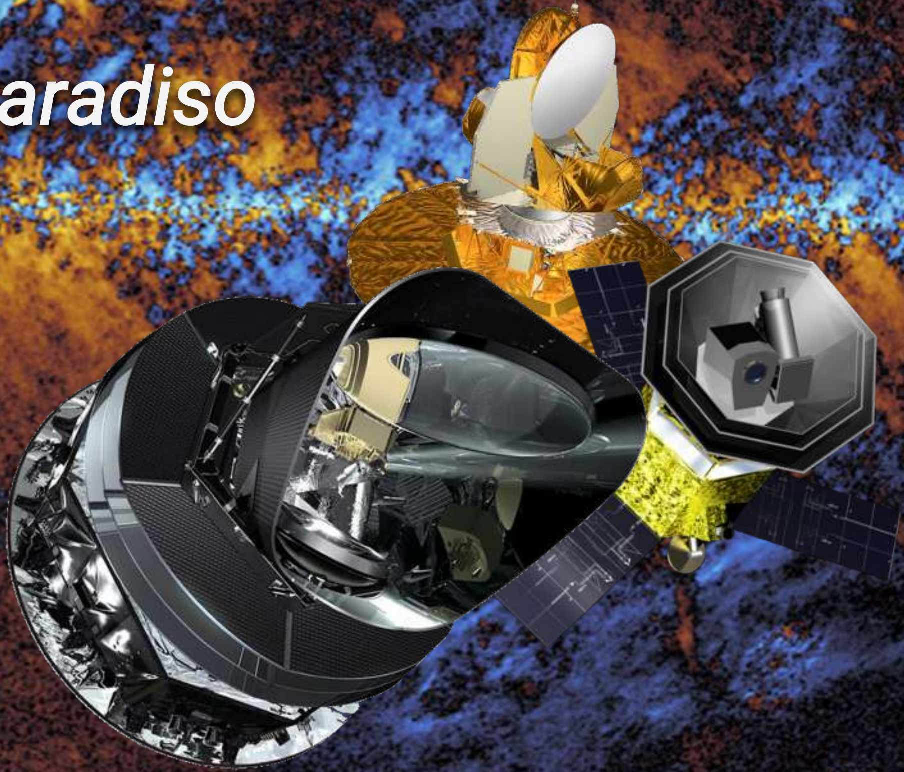
European Commission

Beyond PLANCK

CMB analysis with end-to-end error propagation: Likelihood and Cosmological Parameter

Simone Paradiso



BeyondPlanck online release conference, November 18-20, 2020

This project has received funding from the European Union’s Horizon 2020 research and innovation programme under grant agreement No 776282

BeyondPlanck dataset

- BeyondPlanck main processing:

$$\left(\mathsf{S}^{-1} + \sum_{\nu} \mathsf{M}_{\nu}^{t}\mathsf{B}_{\nu}^{t}\mathsf{N}_{\nu}^{-1}\mathsf{B}_{\nu}\mathsf{M}_{\nu}\right)\boldsymbol{a} = \sum_{\nu} \mathsf{M}_{\nu}^{t}\mathsf{B}_{\nu}^{t}\mathsf{N}_{\nu}^{-1}\boldsymbol{m}_{\nu} + \sum_{\nu} \mathsf{M}_{\nu}^{t}\mathsf{B}_{\nu}^{t}\mathsf{N}_{\nu}^{-1/2}\eta_{\nu} + \mathsf{S}^{-1/2}\eta_{0}.$$

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$S = 0$

Low resolution resampling → $4 \cdot 10^4$ CMB samples at NSIDE=32

BeyondPlanck dataset

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$S \neq 0$

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High resolution resampling → 900 CMB samples at full resolution, with spatial prior $\neq 0$

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Low resolution resampling → $4 \cdot 10^4$ CMB samples at NSIDE=32

High resolution resampling → 900 CMB samples at full resolution, with spatial prior $\neq 0$

Provided a set of samples drawn from the full data posterior distribution → complete end-to-end uncertainty propagation of the sampled parameters

BeyondPlanck Likelihood

- Overall coverage of the multipoles from $\ell = 2$ up to $\ell = 600$ in TT spectrum.
- Information from polarization E modes, and cross-correlation TE, from multipoles in the range $[2 - 8]$.

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Low-ℓ pixel-based Likelihood

TT-TE-EE in $2 \leq \ell \leq 8$

$$P(C_\ell | \hat{s}_{CMB}) \propto \frac{e^{-\frac{1}{2}\, \hat{s}^t{}_{CMB} (S(C_\ell)+N)^{-1}\, \hat{s}_{CMB}}}{|S(C_\ell) + N|^{\frac{1}{2}}}$$

BeyondPlanck Likelihood

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High-ℓ likelihood based upon Gaussianized Blackwell-Rao estimator

TT only in $9 \leq \ell \leq 600$

Beyond PLANCK

BP low-ℓ likelihood

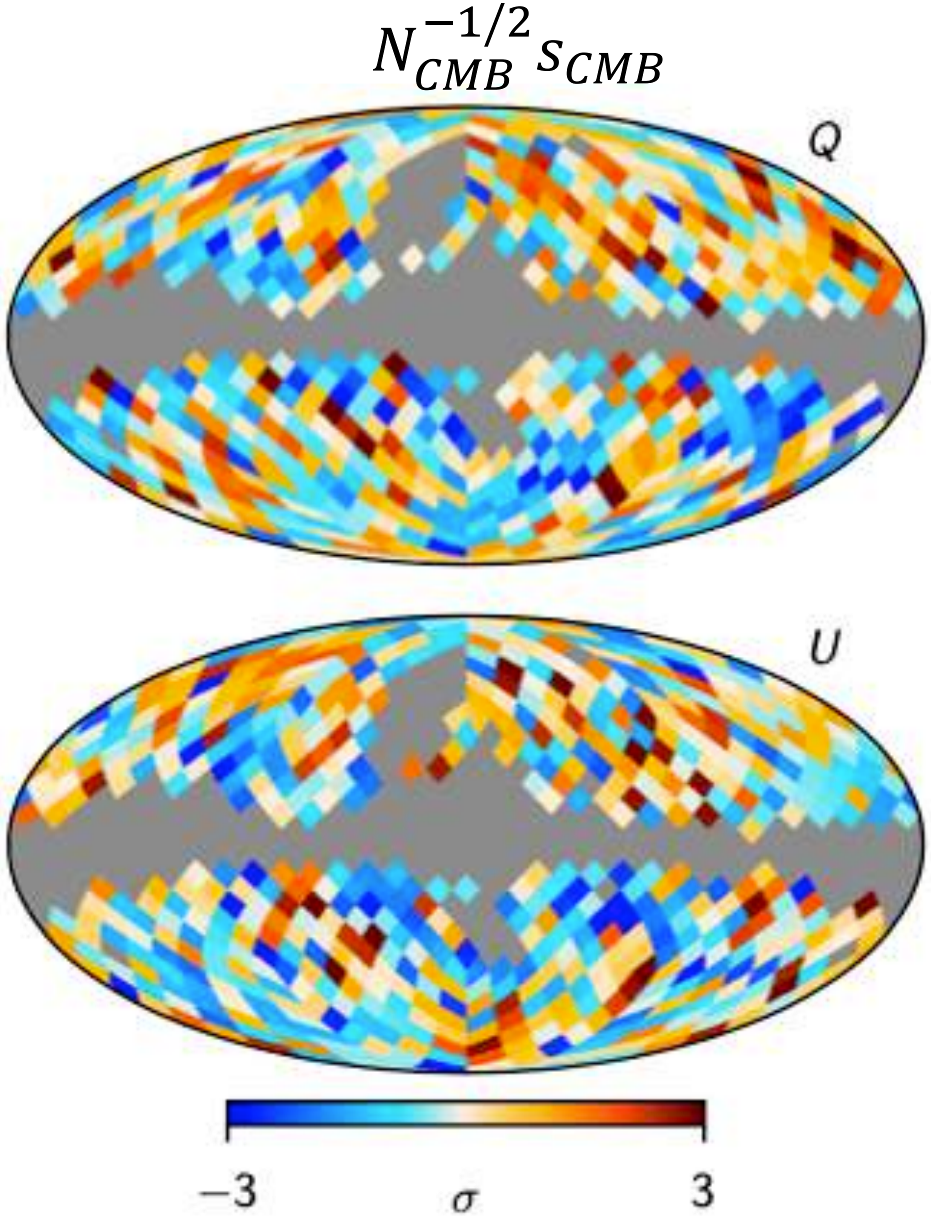
- Direct CMB map and NCVM estimation from $\sim 4 \cdot 10^4$ low resolution samples.

$$\hat{\mathbf{s}}_{CMB} = \langle \mathbf{s}^i_{CMB} \rangle$$

$$\mathrm{N} = \langle \left(\mathbf{s}^i_{CMB} - \mathbf{s}_{CMB}\right)\left(\mathbf{s}^i_{CMB} - \mathbf{s}_{CMB}\right)^t \rangle$$

- Karhunen-Loève compression to isolate only significant modes.

Filter out S/N eigenmodes under a threshold 10^{-6} and multipoles below $\ell_t = 8$.



Colombo et al. 2020

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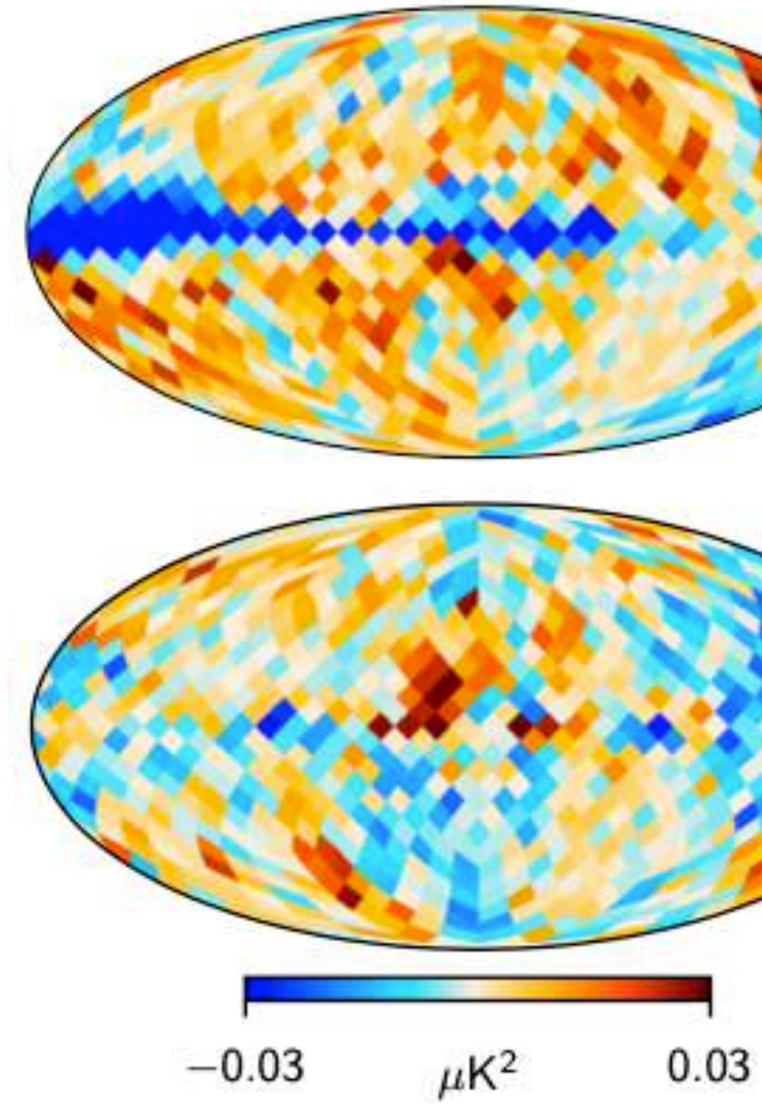

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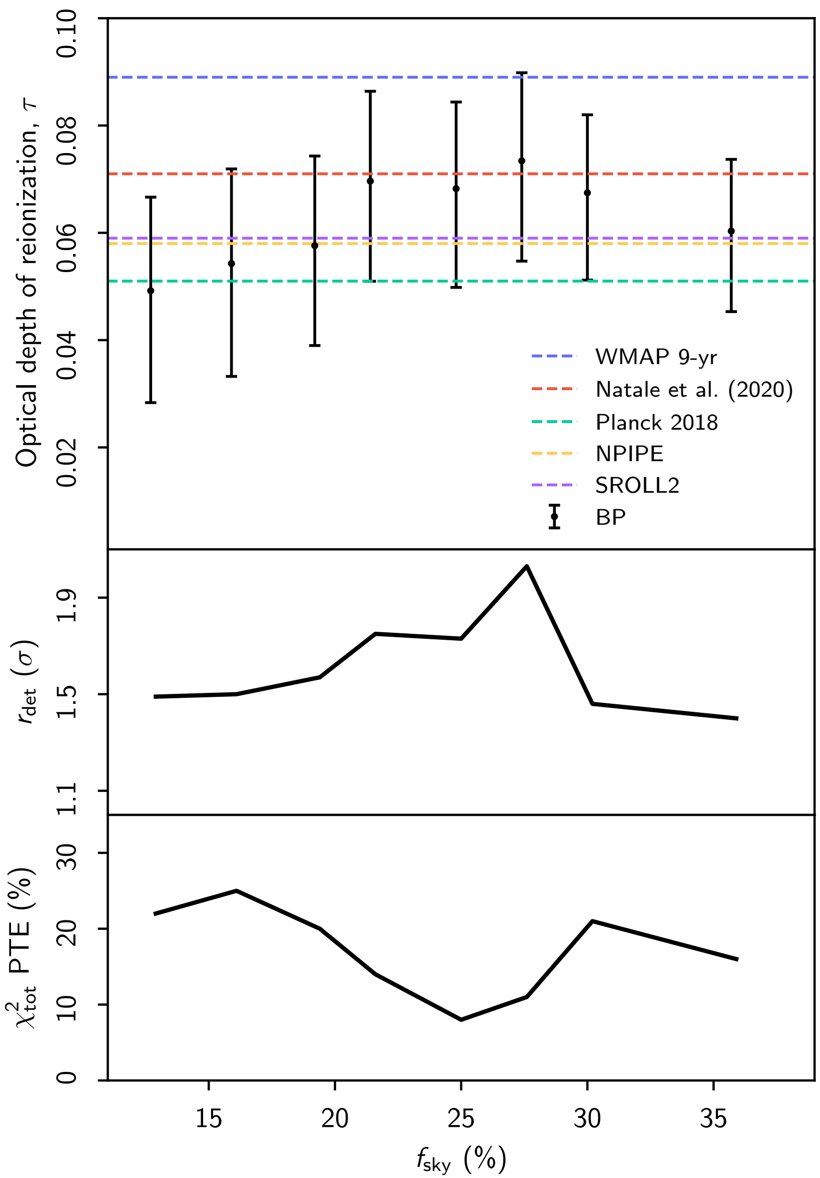

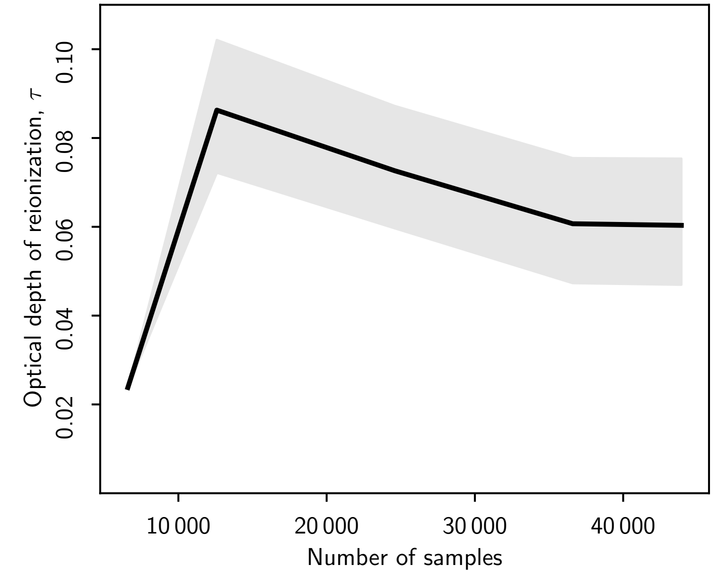

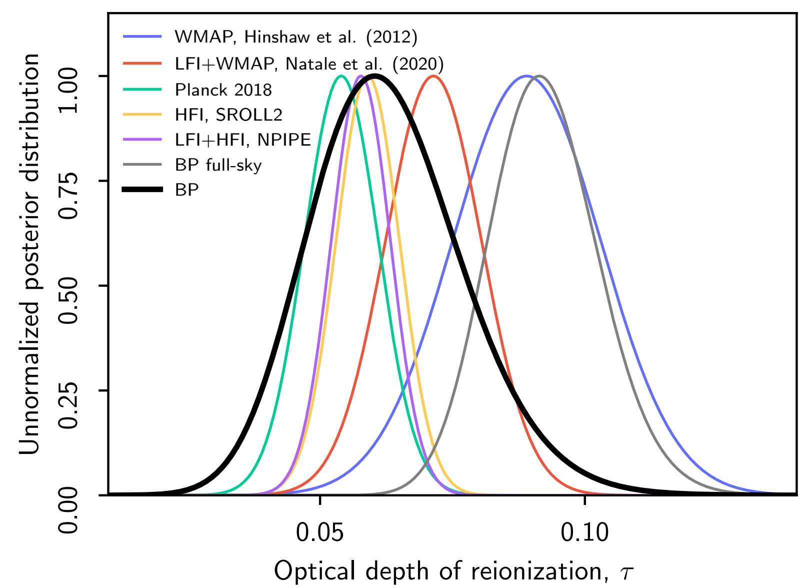

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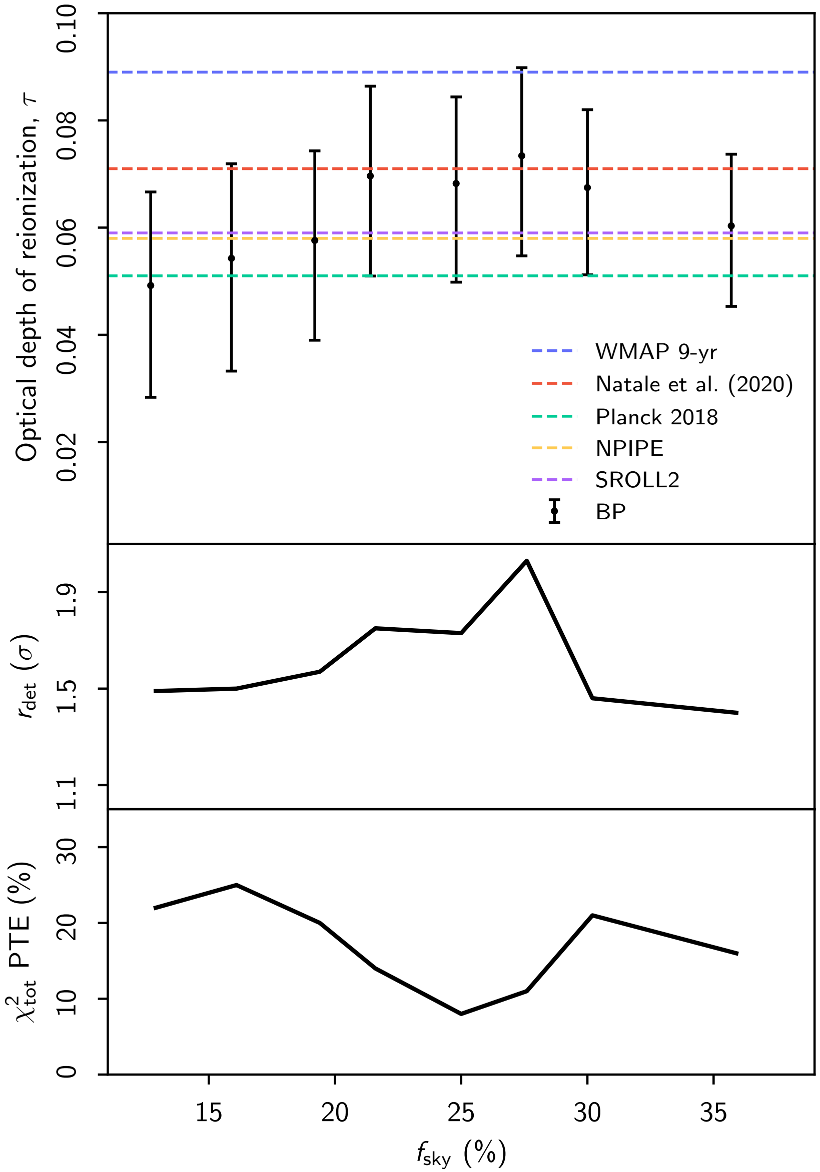

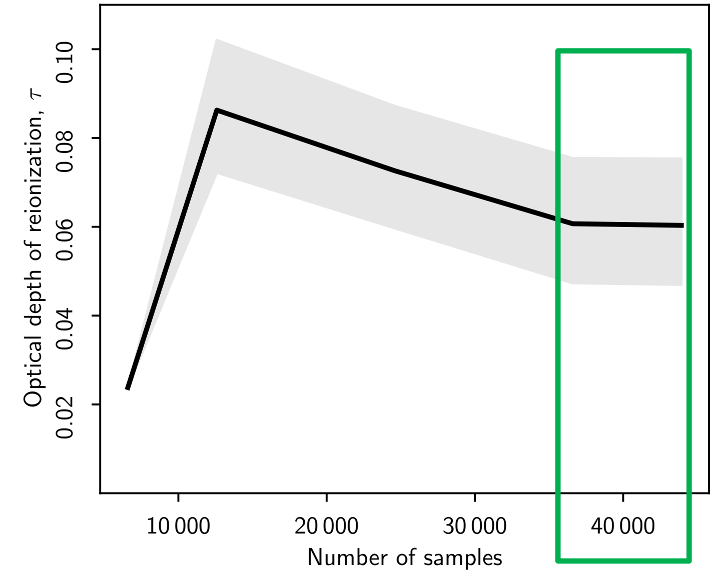

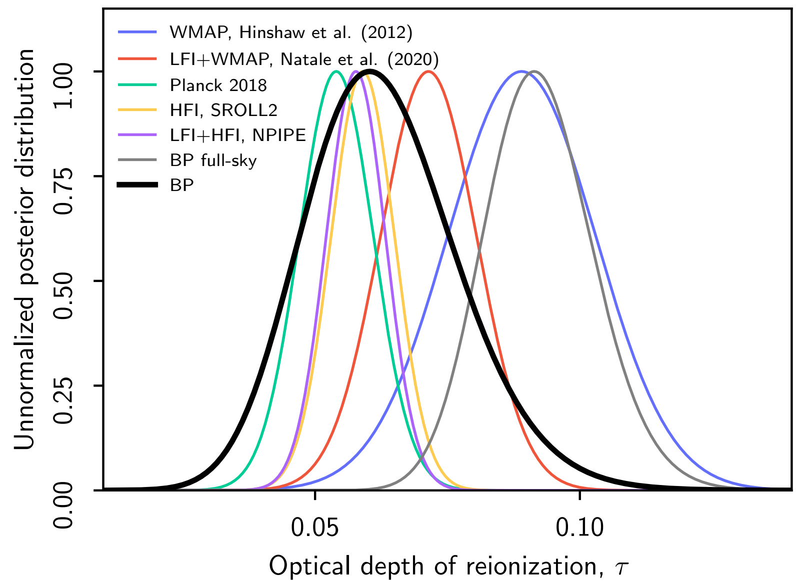

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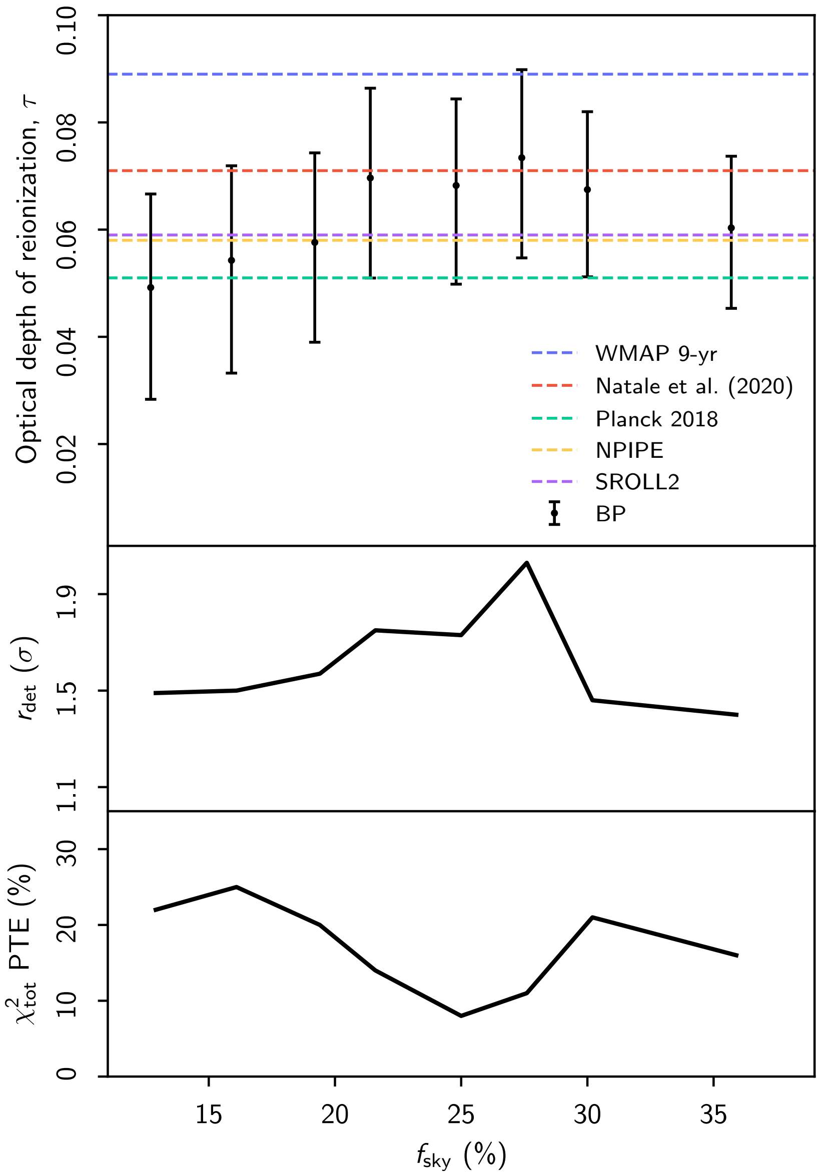

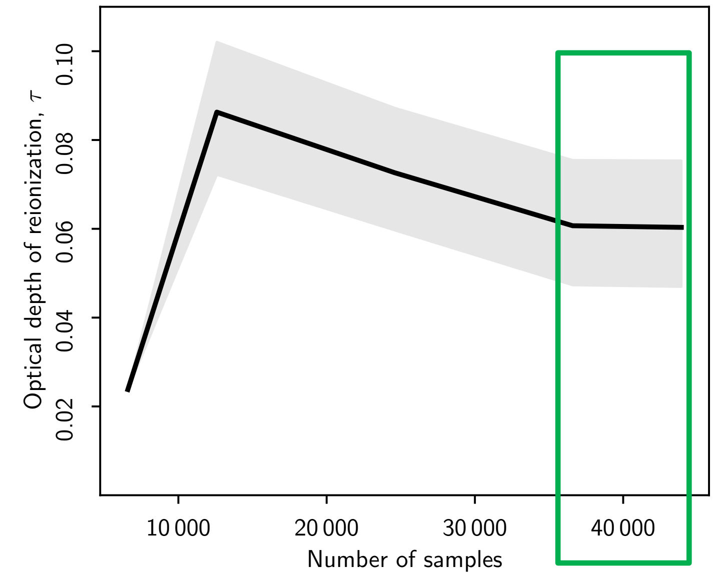

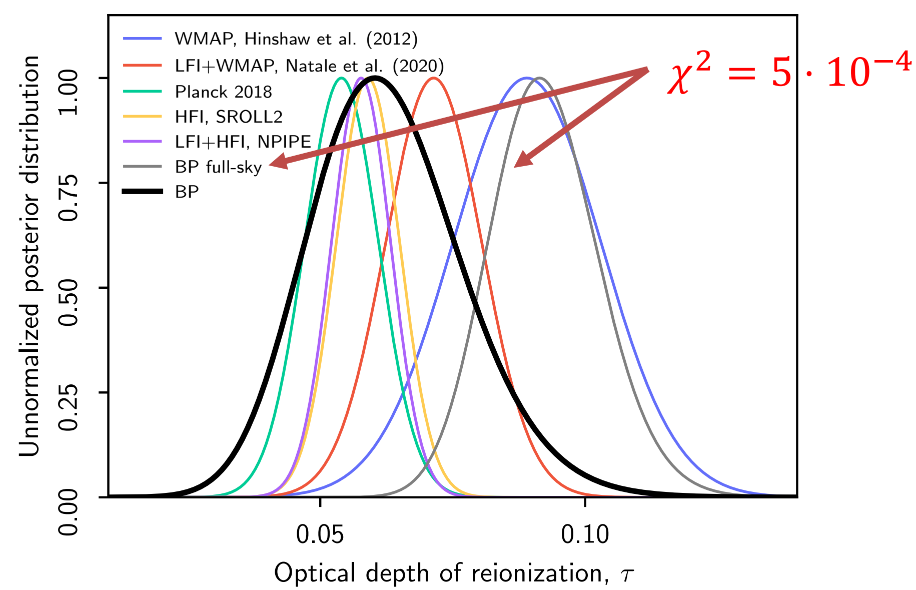

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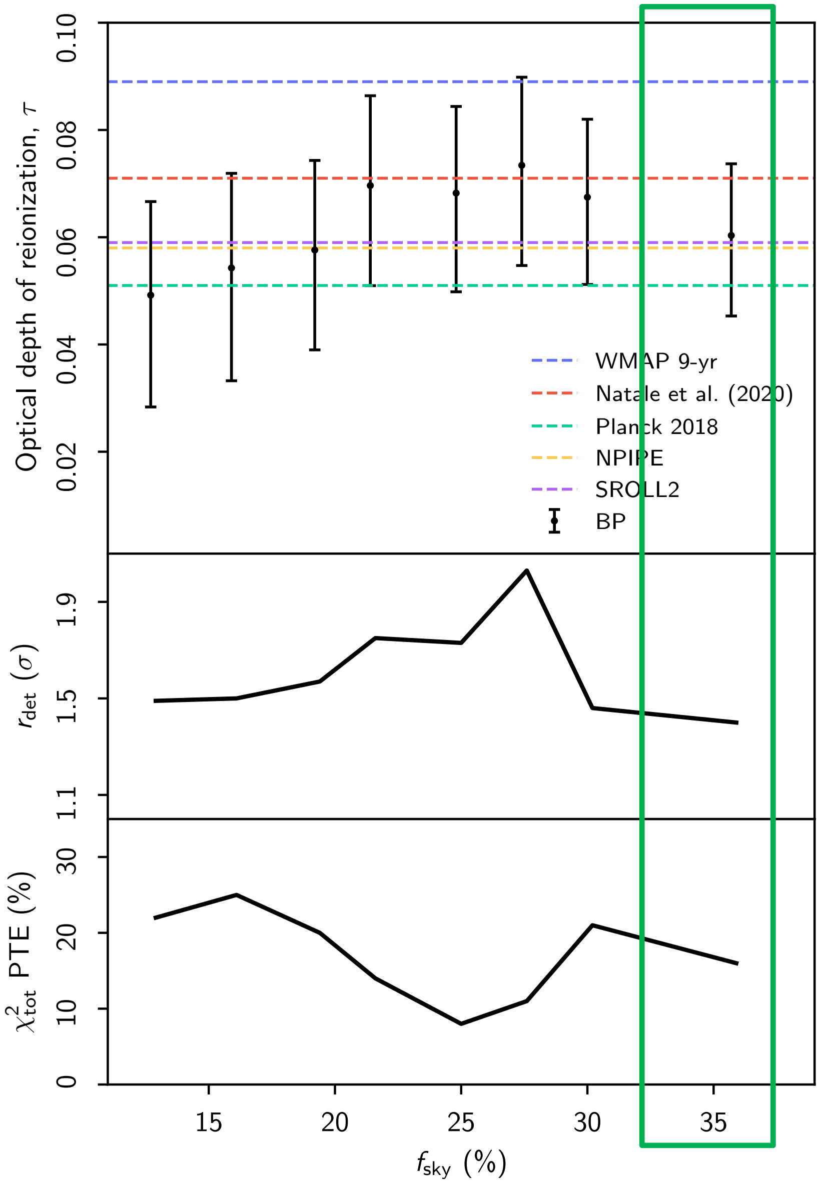

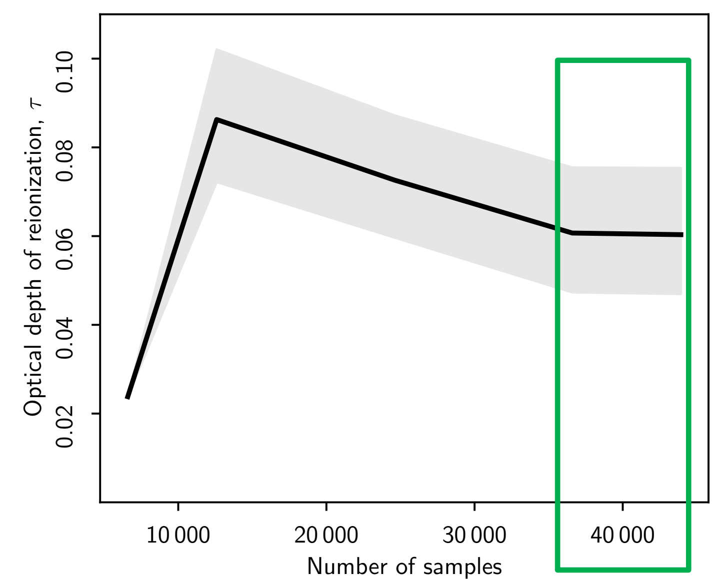

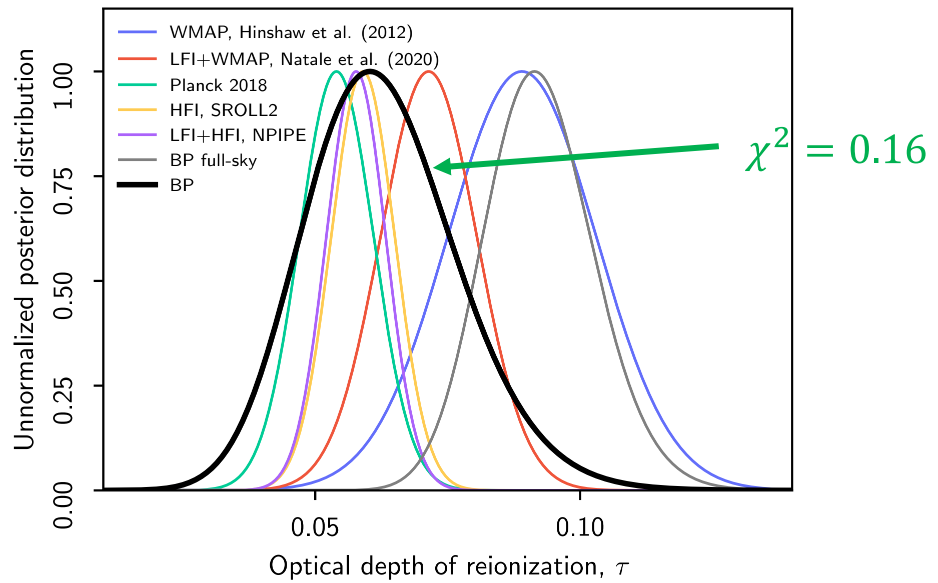

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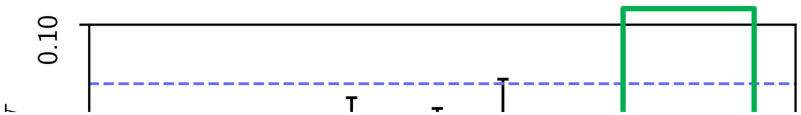
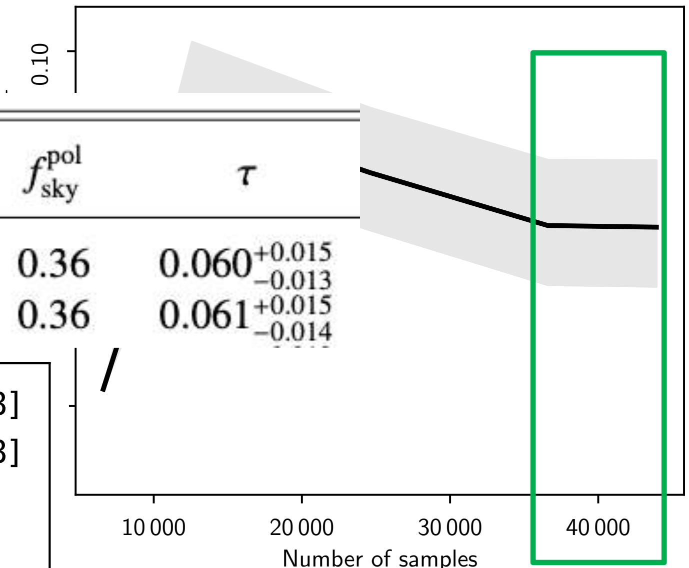
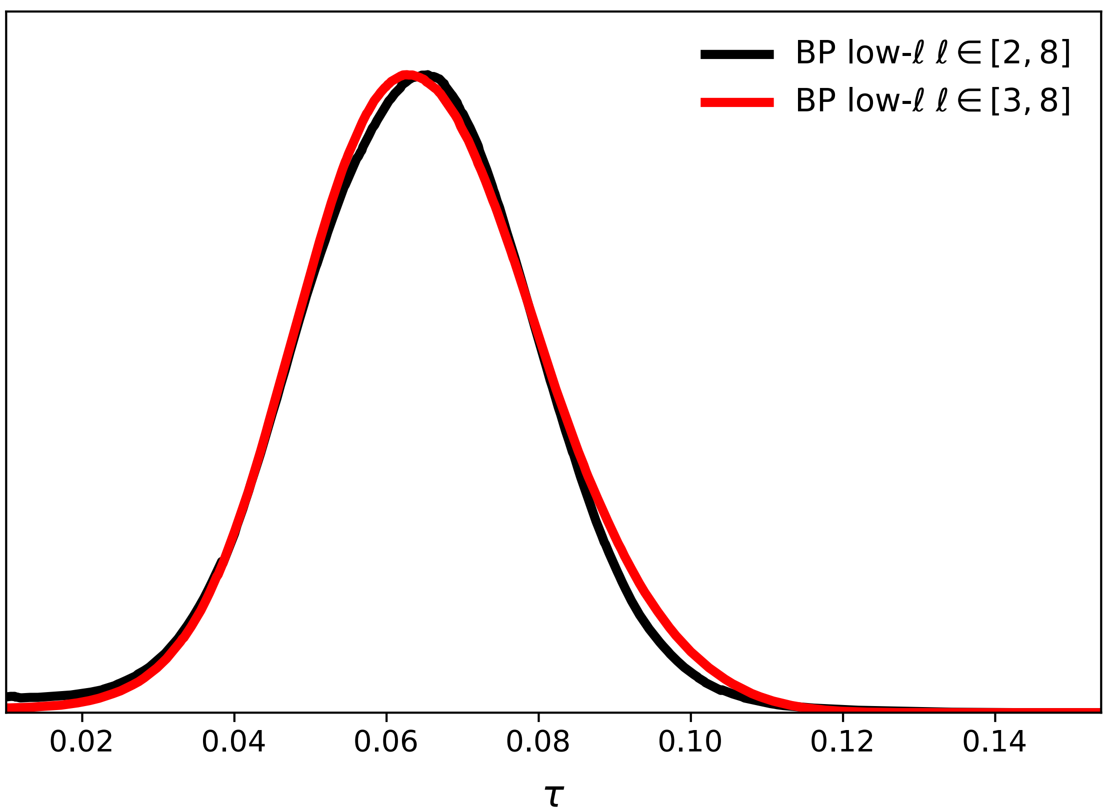
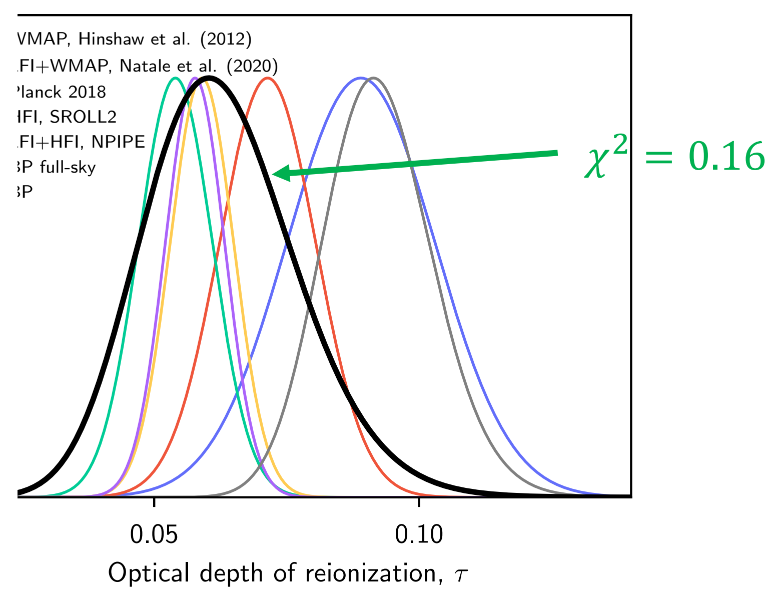




Analysis Name	Data Sets	$f_{\rm sky}^{\rm pol}$	τ
BeyondPlanck, $\ell = 2$–8	LFI, *WMAP Ka–V*	0.36	$0.060^{+0.015}_{-0.013}$
BeyondPlanck, $\ell = 3$–8	LFI, *WMAP Ka–V*	0.36	$0.061^{+0.015}_{-0.014}$

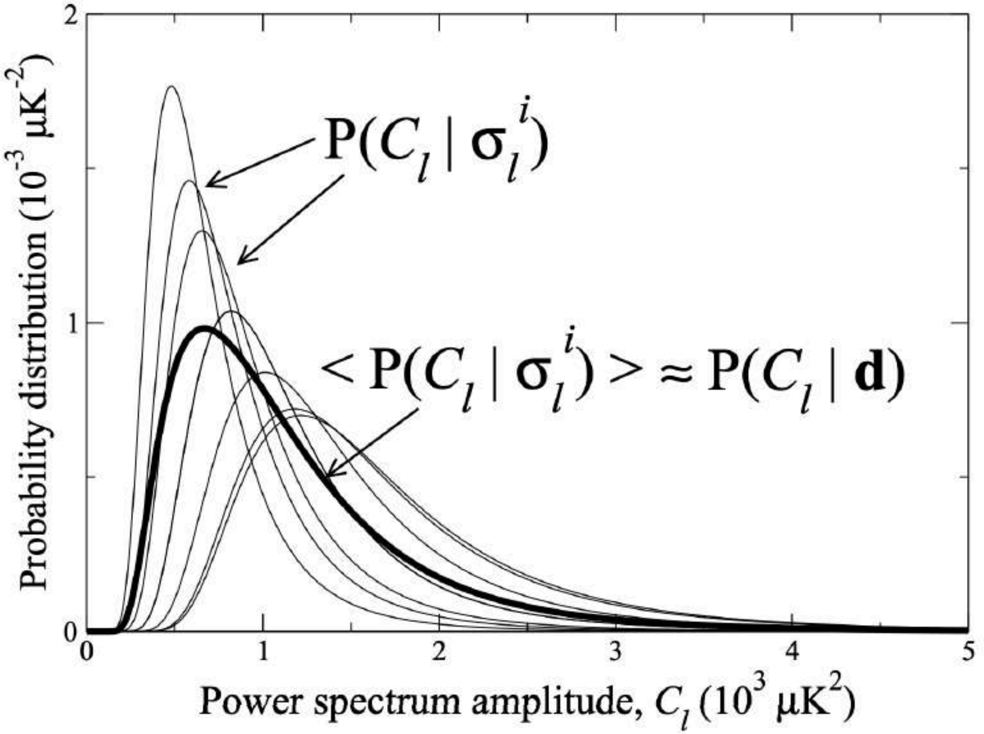




BP high-ℓ likelihood

- Gaussianized Blackwell-Rao (Rudjord et al. 2009) estimator from 900 high resolution resampled CMB maps.
- Low S/N ratio in polarization at $\ell > 10$ → Only temperature

Chu et al. 2005



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$$P(C_\ell \,|\, \boldsymbol{d}) \approx \frac{1}{N_G} \sum_{i=1}^{N_G} P(C_\ell \,|\, \sigma_\ell^i)$$

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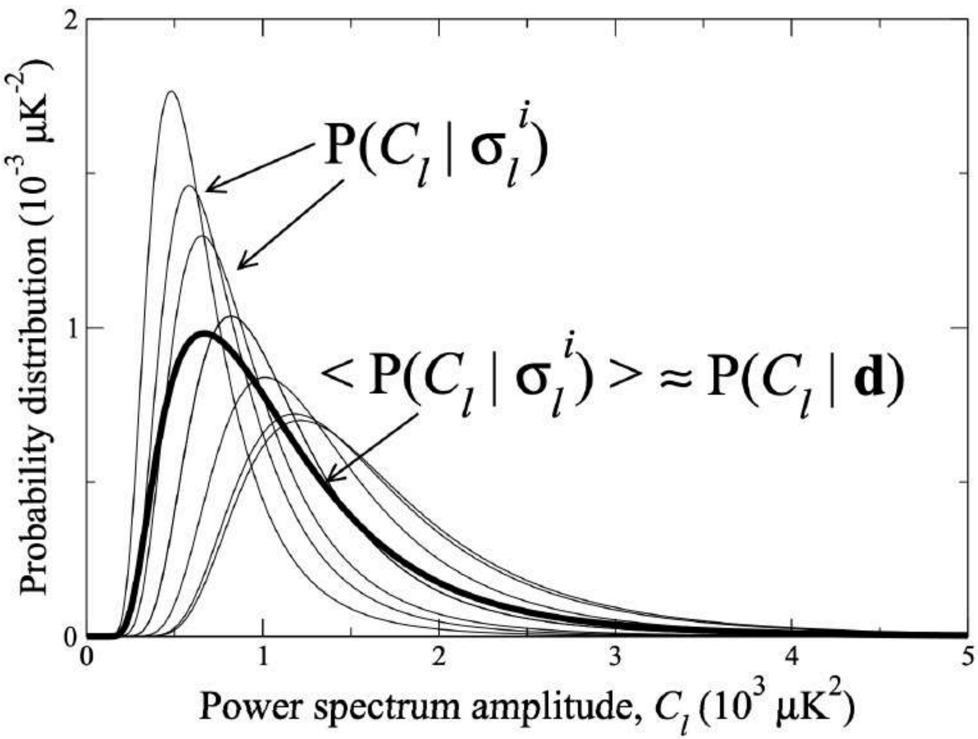

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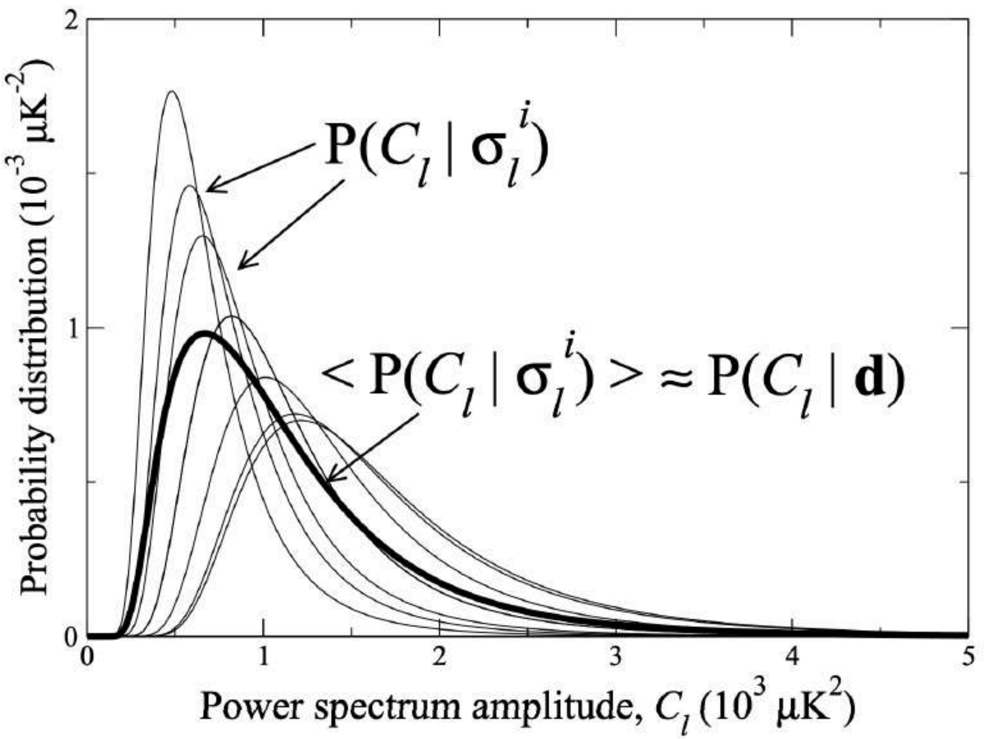
$$P(C_\ell \,|\, \boldsymbol{d}) = \left(\prod_\ell \frac{\partial C_\ell}{\partial x_\ell} \right)^{-1} P(\boldsymbol{x} \,|\, \boldsymbol{d})$$

$$P(\boldsymbol{x} \,|\, \boldsymbol{d}) \approx e^{-\frac{1}{2}(\boldsymbol{x}-\mu)^T \mathrm{C}^{-1} (\boldsymbol{x}-\mu)}$$

Chu et al. 2005



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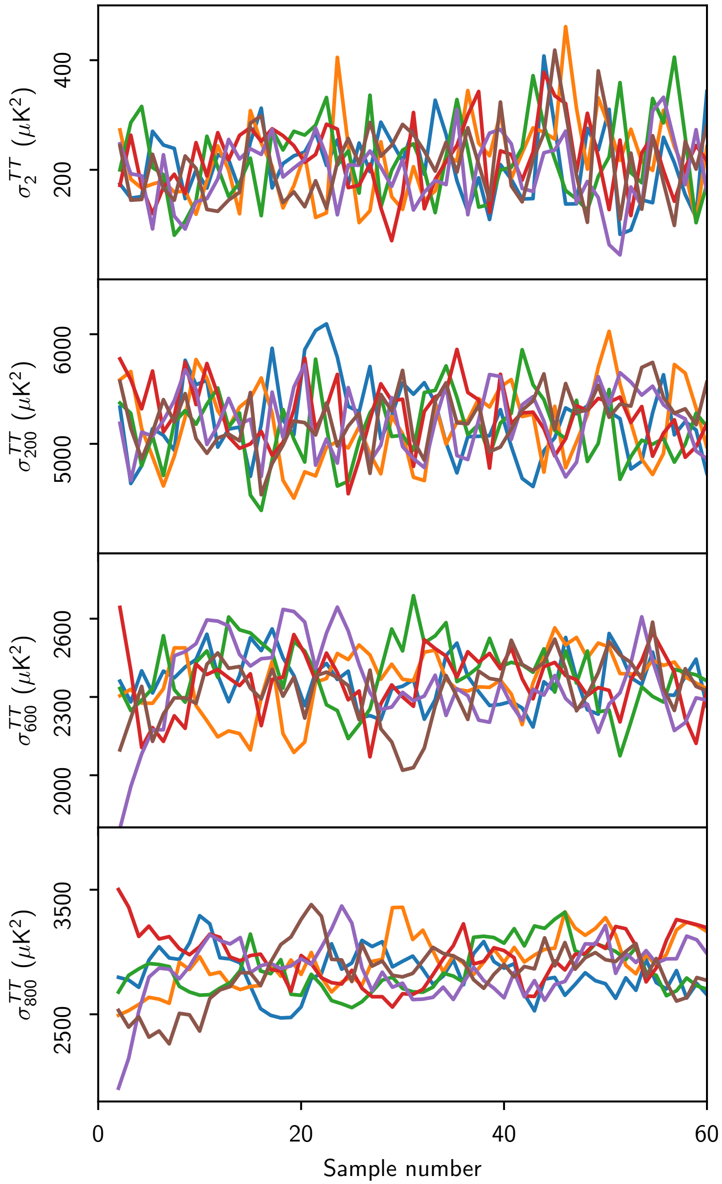

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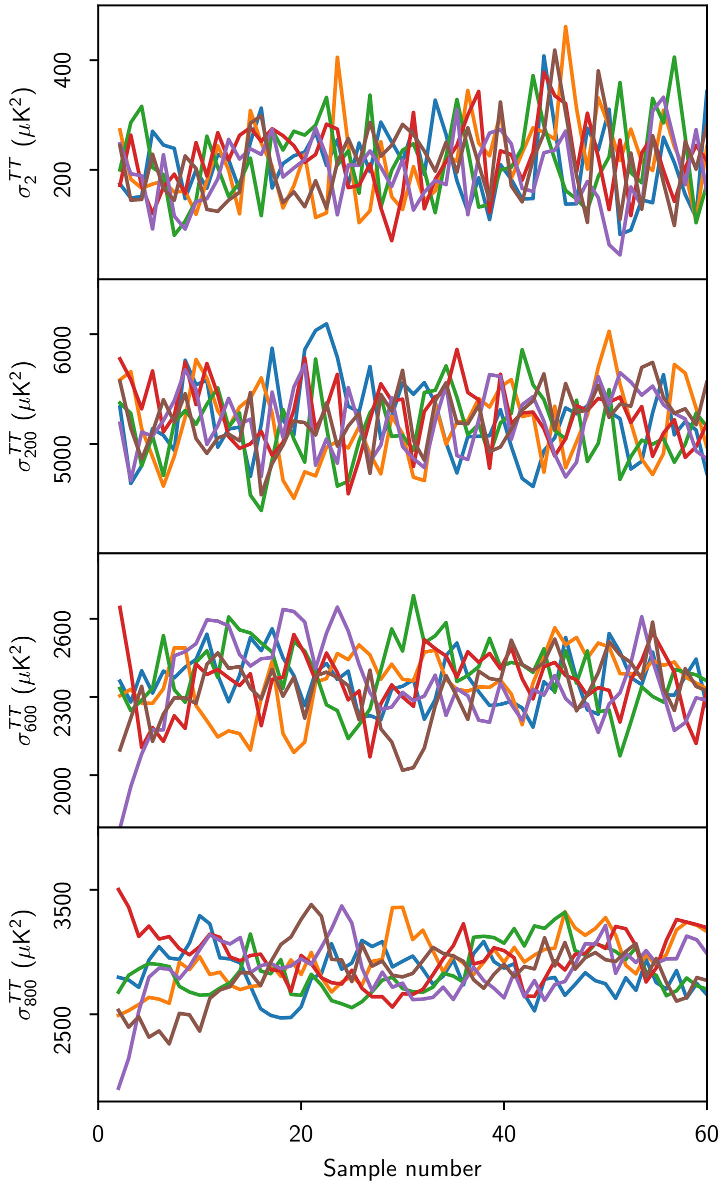
longer correlation length, but still good convergency



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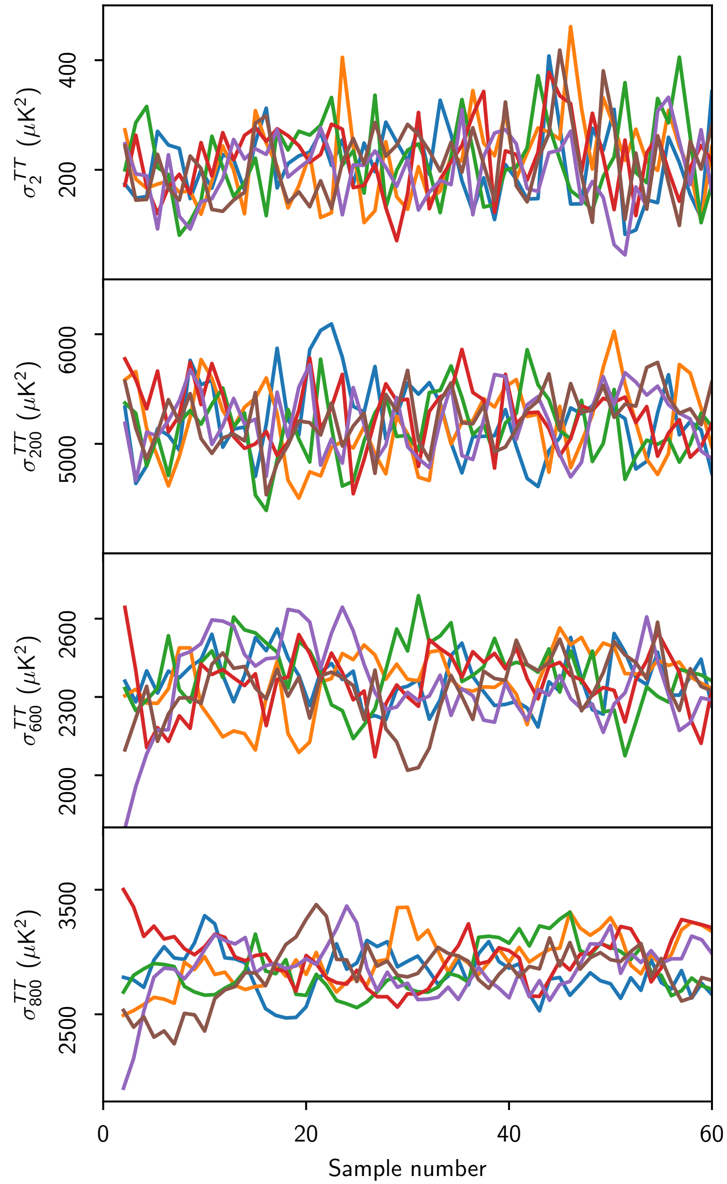

Beyond PLANCK

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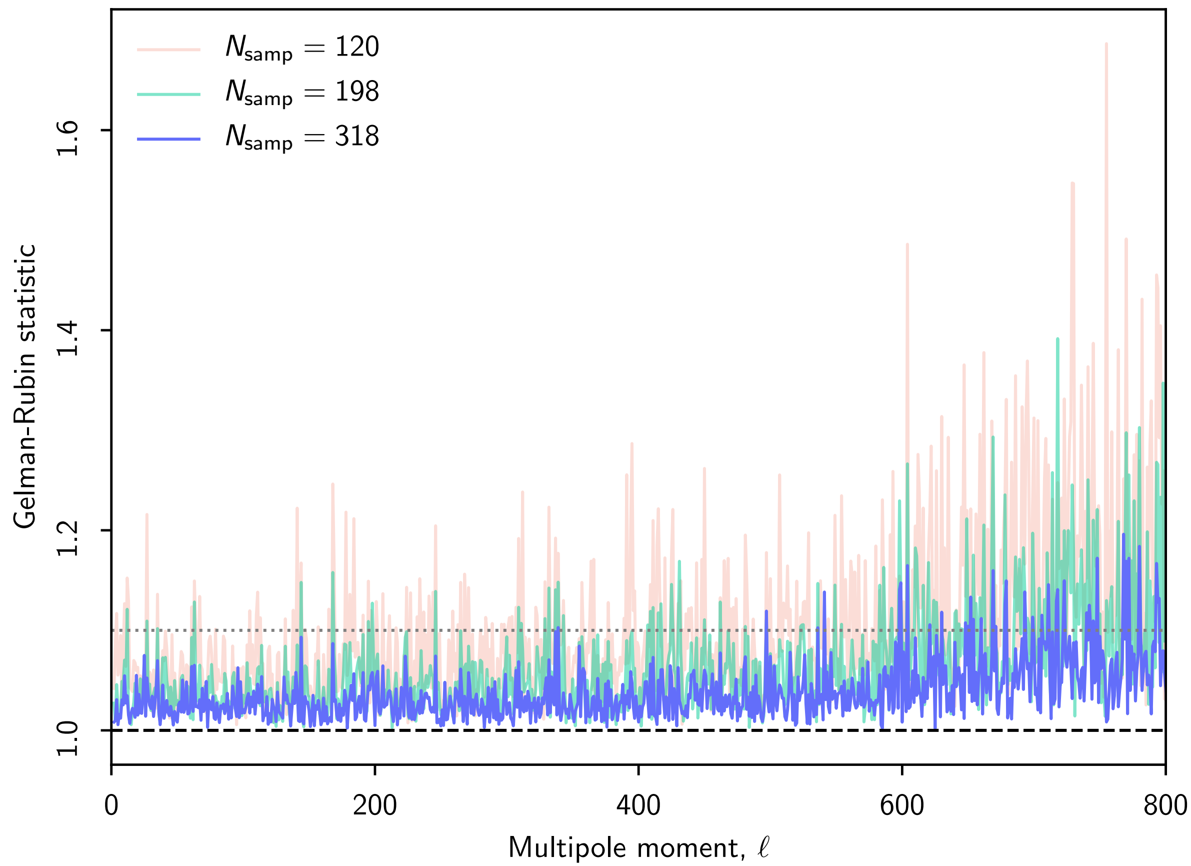
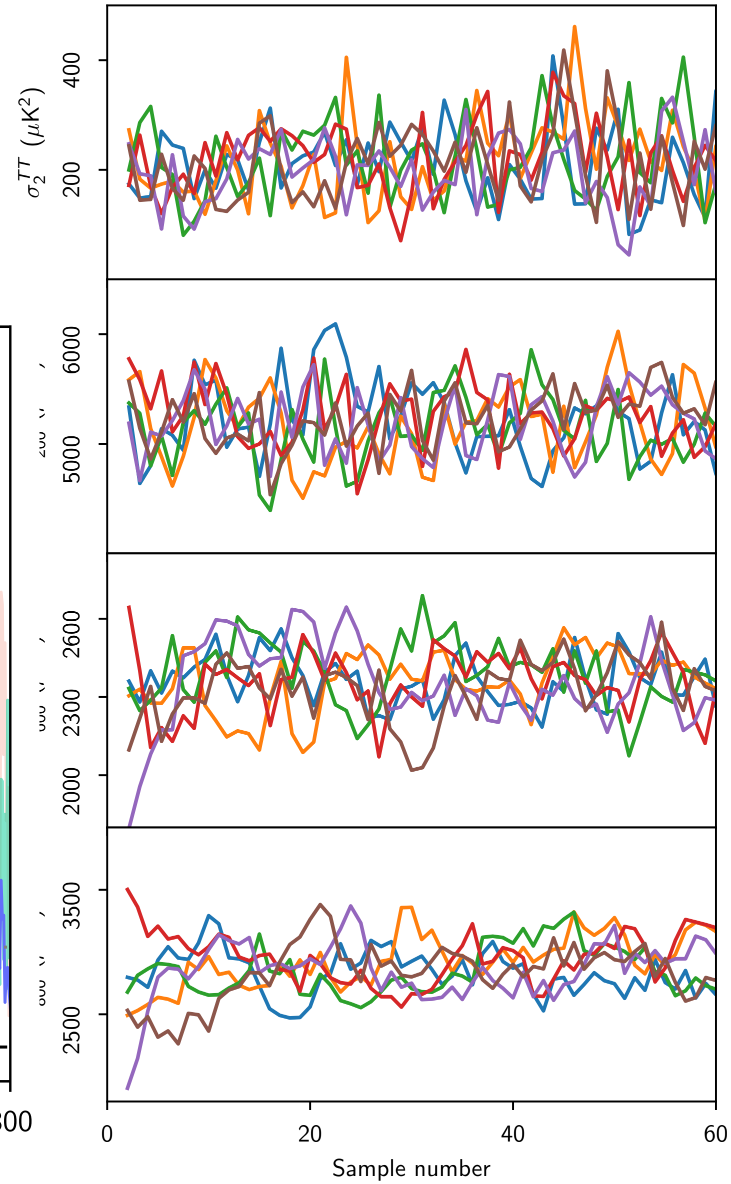


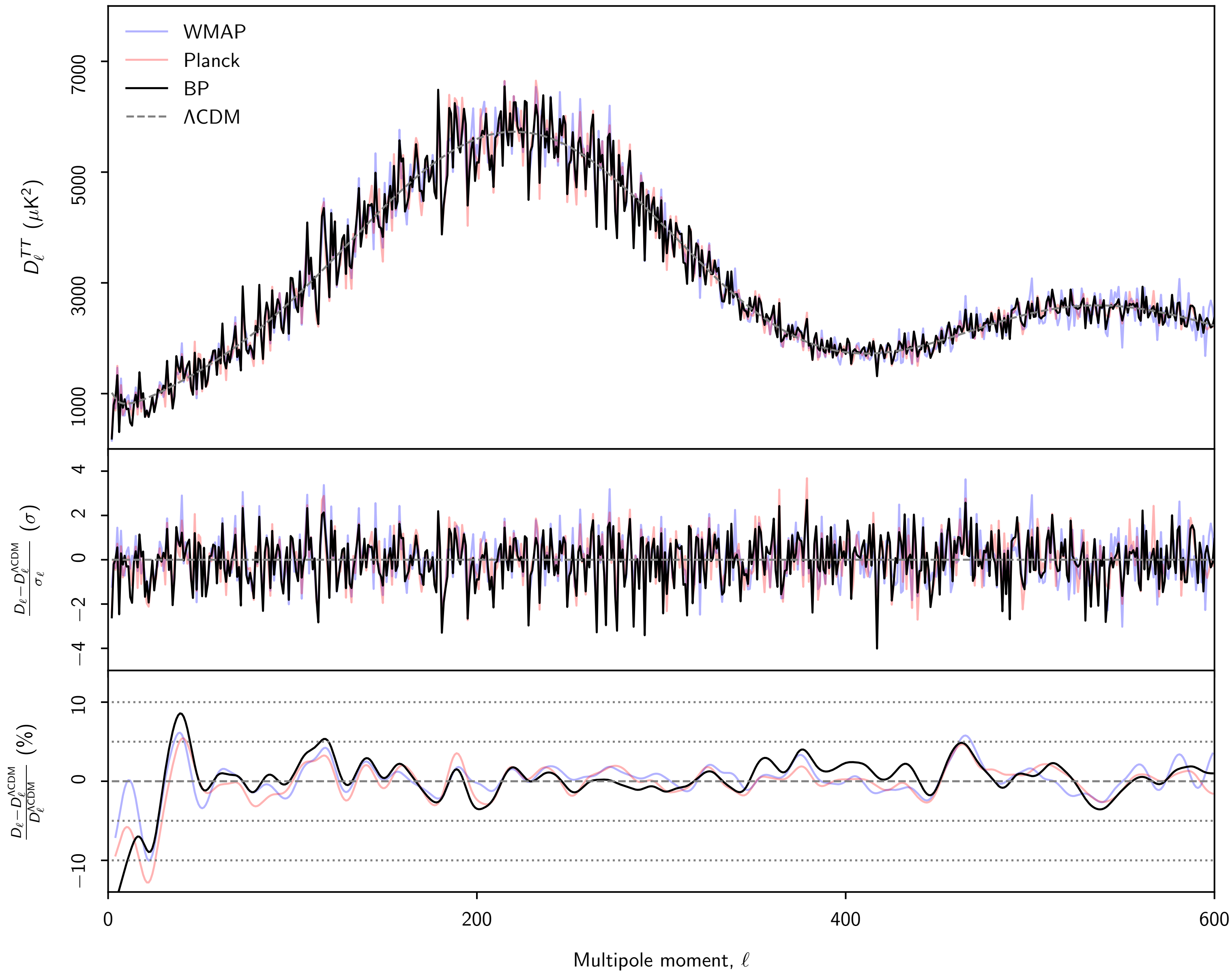


BP high-ℓ likelihood

Stable parameter estimates up to $\ell = 600$

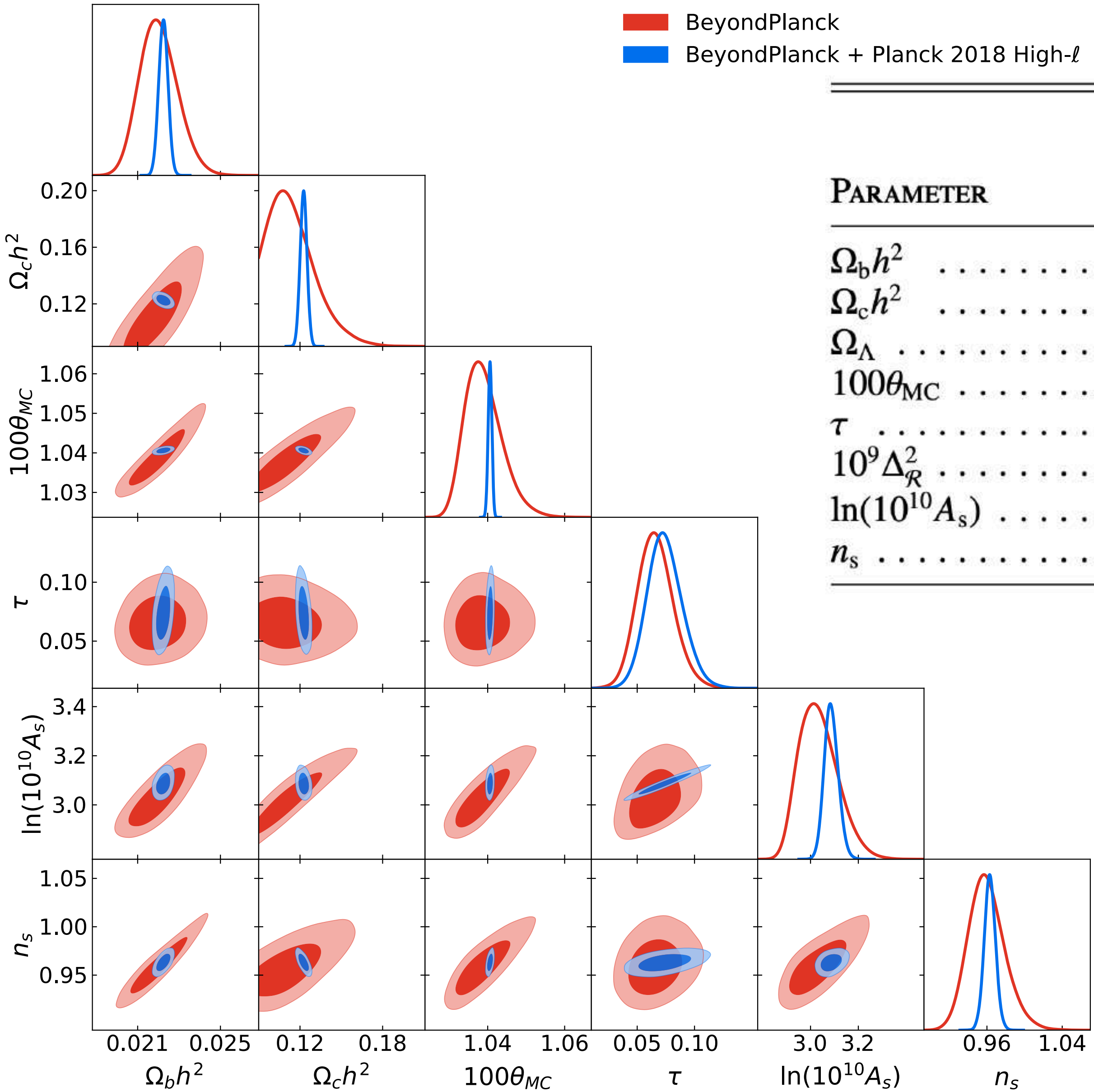
Parameter	BeyondPlanck GBR $\ell_{max} = 400$	BeyondPlanck GBR $\ell_{max} = 600$	Δ
$\Omega_b h^2$	0.0229 ± 0.0018	0.0227 ± 0.0013	0.1σ
$\Omega_c h^2$	0.129 ± 0.028	0.116 ± 0.018	0.5σ
$100\theta_{MC}$	1.049 ± 0.011	1.041 ± 0.006	0.7σ
$A_s e^{-2\tau}$	2.01 ± 0.26	1.85 ± 0.15	0.6σ
n_s	1.011 ± 0.054	0.980 ± 0.036	0.6σ

BP high-ℓ likelihood



Colombo et al. 2020

Full BeyondPlanck likelihood results



	BeyondPlanck	
Parameter	$\ell \leq 600$	$+Planck\ \ell > 600$
$\Omega_b h^2$	0.02202 ± 0.00091	0.02224 ± 0.00022
$\Omega_c h^2$	0.115 ± 0.017	0.1224 ± 0.0025
Ω_Λ	...	...
$100\theta_{MC}$	1.0390 ± 0.0049	1.04061 ± 0.00048
τ	0.066 ± 0.016	0.074 ± 0.015
$10^9\Delta^2_{\mathcal{R}}$	...	...
$\ln(10^{10}A_s)$	3.035 ± 0.080	3.087 ± 0.029
n_s	0.960 ± 0.020	0.9632 ± 0.0060

Full BeyondPlanck likelihood results

Parameter	BeyondPlanck $\ell \leq 600$	BeyondPlanck $+Planck\ \ell > 600$	*Planck* 2018 Estimate	*Planck* 2018 $\Delta(\sigma)$	*WMAP* Estimate	*WMAP* $\Delta(\sigma)$
$\Omega_b h^2$	0.02202 ± 0.00091	0.02224 ± 0.00022	0.02237 ± 0.00015	-0.4	0.02243 ± 0.00050	-0.5
$\Omega_c h^2$	0.115 ± 0.017	0.1224 ± 0.0025	0.1200 ± 0.0012	-0.3	0.1147 ± 0.0051	0
Ω_Λ	...	...	...	...	0.721 ± 0.025	...
$100\theta_{MC}$	1.0390 ± 0.0049	1.04061 ± 0.00048	1.04092 ± 0.00031	-0.4	...	...
τ	0.066 ± 0.016	0.074 ± 0.015	0.054 ± 0.007	0.8	0.089 ± 0.0014	-1.5
$10^9 \Delta^2_{\mathcal{R}}$	...	...	...	...	2.41 ± 0.10	...
$\ln(10^{10} A_s)$	3.035 ± 0.080	3.087 ± 0.029	3.044 ± 0.014	-0.1	...	...
n_s	0.960 ± 0.020	0.9632 ± 0.0060	0.9649 ± 0.0042	-0.3	0.972 ± 0.013	-0.6

Full BeyondPlanck likelihood results

Parameter	BeyondPlanck $\ell \leq 600$	BeyondPlanck $+Planck\ \ell > 600$	*Planck* 2018 Estimate	*Planck* 2018 $\Delta\,(\sigma)$	*WMAP* Estimate	*WMAP* $\Delta\,(\sigma)$
$\Omega_b h^2$	0.02202 ± 0.00091	0.02224 ± 0.00022	0.02237 ± 0.00015	-0.4	0.02243 ± 0.00050	-0.5
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Full BeyondPlanck likelihood results

Only LFI and WMAP → major contribution to larger uncertainties

	BeyondPlanck		Planck 2018		WMAP	
Parameter	$\ell \leq 600$	+*Planck* $\ell > 600$	Estimate	$\Delta\,(\sigma)$	Estimate	$\Delta\,(\sigma)$
$\Omega_b h^2$	0.02202 ± 0.00091	0.02224 ± 0.00022	0.02237 ± 0.00015	-0.4	0.02243 ± 0.00050	-0.5
$\Omega_c h^2$	0.115 ± 0.017	0.1224 ± 0.0025	0.1200 ± 0.0012	-0.3	0.1147 ± 0.0051	0
Ω_Λ	...	...	...	...	0.721 ± 0.025	...
$100\theta_{MC}$	1.0390 ± 0.0049	1.04061 ± 0.00048	1.04092 ± 0.00031	-0.4	...	...
τ	0.066 ± 0.016	0.074 ± 0.015	0.054 ± 0.007	0.8	0.089 ± 0.0014	-1.5
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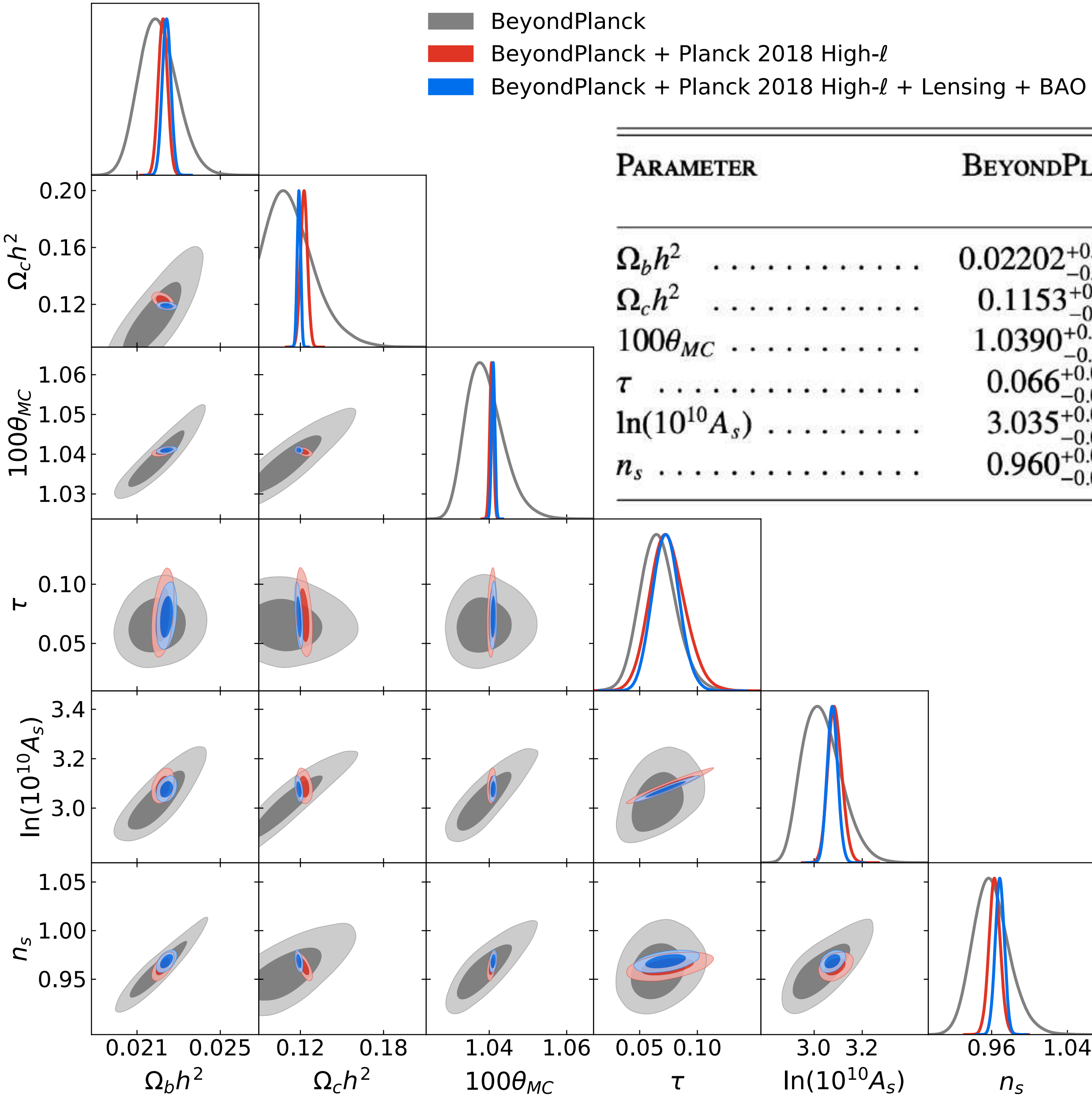
n_s	0.960 ± 0.020	0.9632 ± 0.0060	0.9649 ± 0.0042	-0.3	0.972 ± 0.013	-0.6

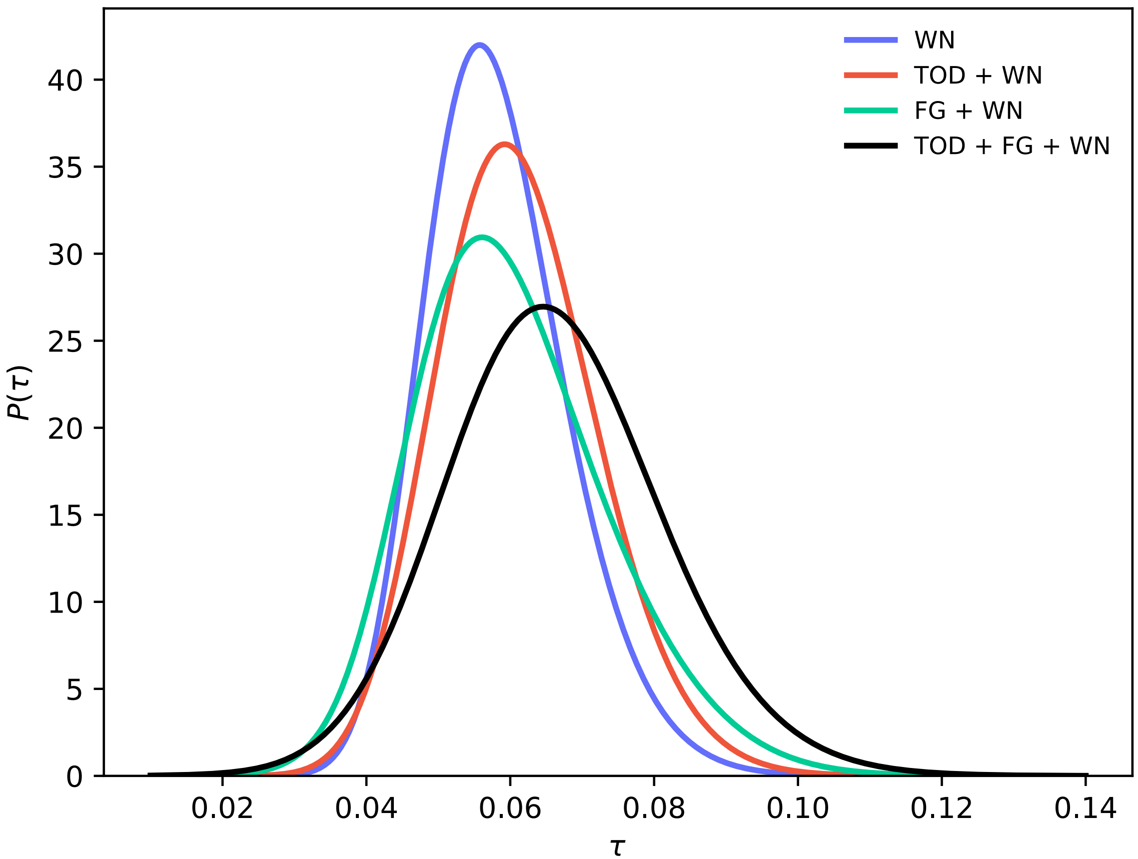
BeyondPlanck + Planck 2018 + Lensing + BAO



Parameter	BeyondPlanck	BeyondPlanck + *Planck*	BeyondPlanck + *Planck* + Lensing + BAO
$\Omega_b h^2$	$0.02202^{+0.00087}_{-0.00099}$	0.02224 ± 0.00022	0.02237 ± 0.00020
$\Omega_c h^2$	$0.1153^{+0.084}_{-0.022}$	0.1226 ± 0.0025	0.1189 ± 0.0012
$100\theta_{MC}$	$1.0390^{+0.0037}_{-0.0056}$	1.04061 ± 0.00048	1.04098 ± 0.00041
τ	$0.066^{+0.014}_{-0.017}$	$0.074^{+0.014}_{-0.016}$	0.072 ± 0.012
$\ln(10^{10}A_s)$	$3.035^{+0.064}_{-0.095}$	$3.087^{+0.027}_{-0.031}$	3.075 ± 0.022
n_s	$0.960^{+0.017}_{-0.021}$	0.9632 ± 0.0060	0.9687 ± 0.0048

End-to-end error propagation

Propagating uncertainties through the whole processing up to cosmological parameter estimation



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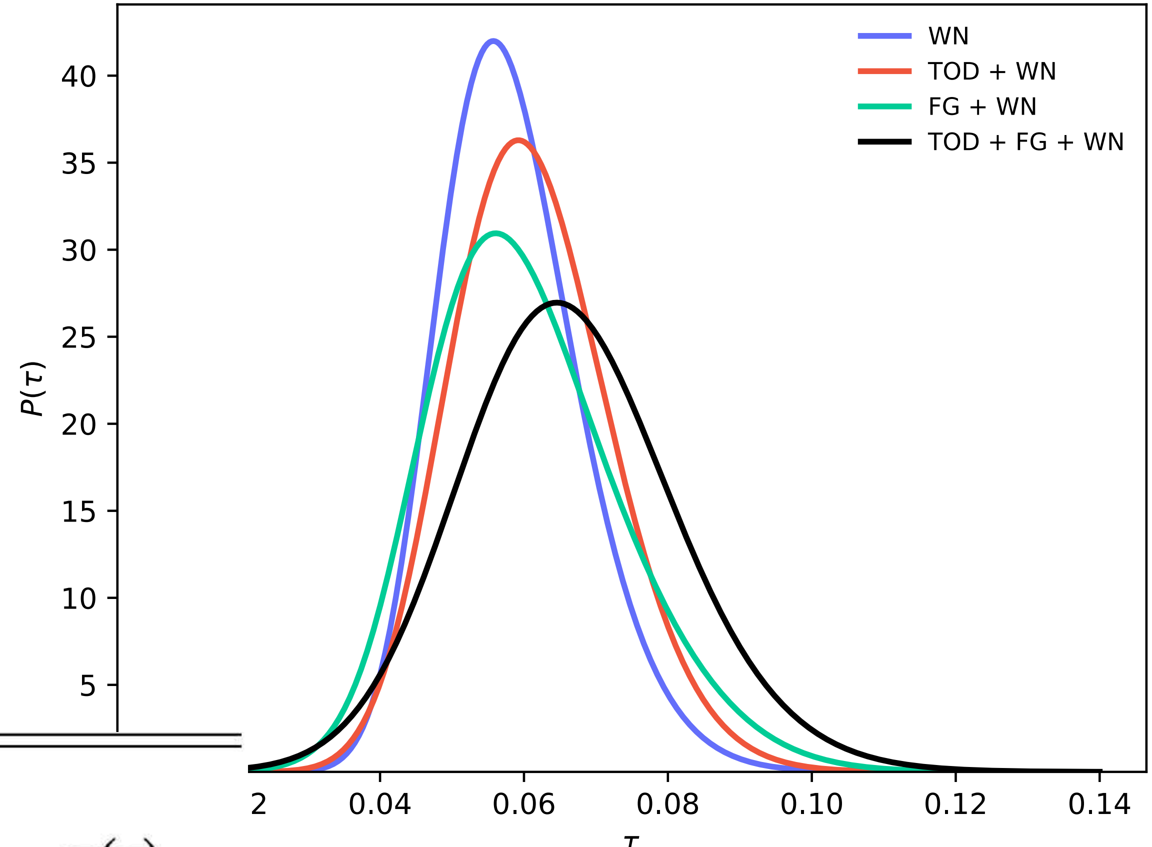


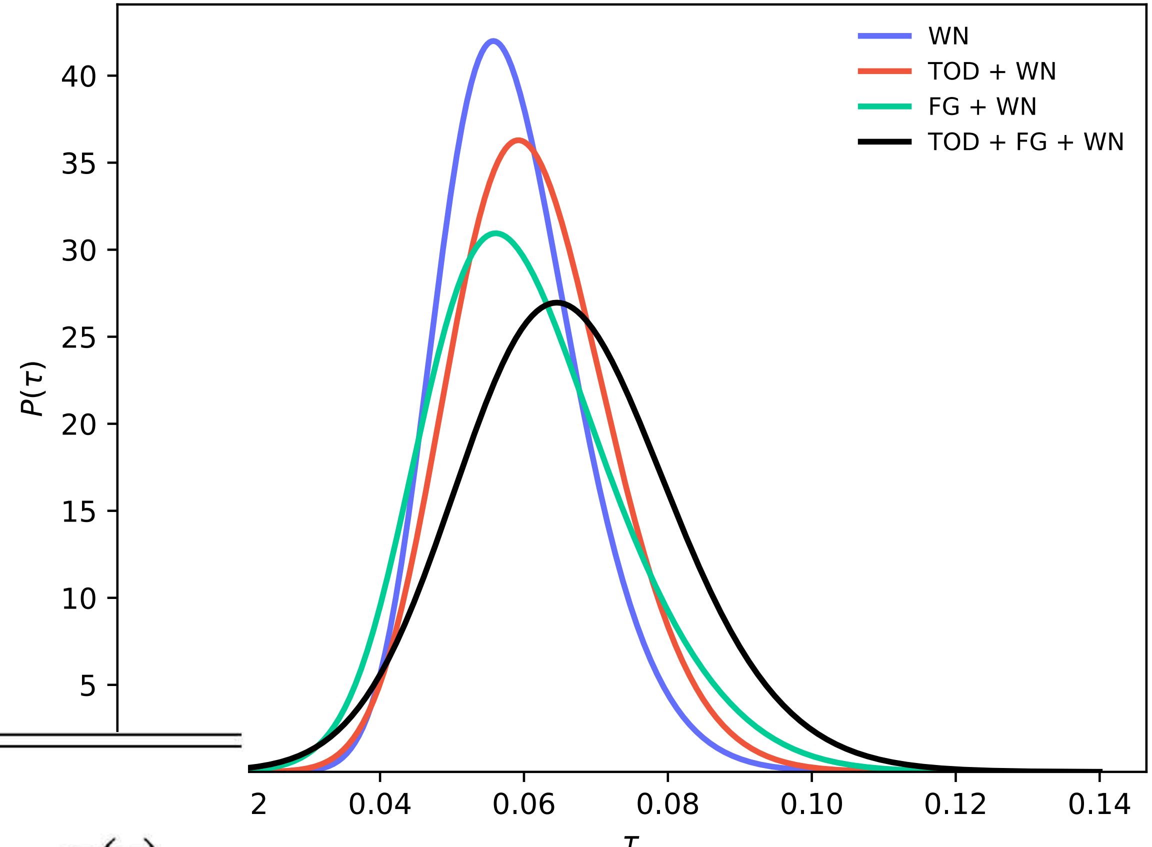
Low-resolution resampling marginalization	$\tau_{\max}$	$\sigma(\tau)$
WN	0.0557	0.0095
TOD + WN	0.0592	0.0109
FG + WN	0.0561	0.0130
TOD + FG + WN	0.0646	0.0148

End-to-end error propagation

Propagating uncertainties through the whole processing up to cosmological parameter estimation

Marginalisation over noise parameters, along with foreground and TOD ones!

$\sigma_{CV} \sim 0.003 - 0.004$



Low-resolution resampling marginalization	τ_{max}	$\sigma(\tau)$
WN	0.0557	0.0095
TOD + WN	0.0592	0.0109
FG + WN	0.0561	0.0130
TOD + FG + WN	0.0646	0.0148

FG+WN coupling leads to error under-estimation!

Summary

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Beyond PLANCK

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- We estimated cosmological parameter within a global Bayesian analysis framework.
- Implemented two likelihoods, a low-ℓ pixel-based for both temperature and polarization, and a GBR based one, for higher TT multipoles.
- Showed the impact of marginalizing over model parameters, in terms of τ posterior uncertainty.

The point was to show how methodology can provide cosmological parameter estimates, along with correctly propagating model parameters uncertainties throughout the analysis pipeline.

The BeyondPlanck collaboration

European Commission

EU-funded institutions

Kristian Joten Andersen
Ragnhild Aurlien
Ranajoy Banerji
Maksym Brilenkov
Hans Kristian Eriksen
Johannes Røsok Eskilt
Marie Kristine Foss
Unni Fuskeland
Eirik Gjerløw
Mathew Galloway
Daniel Herman
Ata Karakci
Håvard Tveit Ihle
Metin San
Trygve Leithe Svalheim
Harald Thommesen
Duncan Watts
Ingunn Kathrine Wehus

Marco Bersanelli
Loris Colombo
Cristian Franceschet
Davide Maino
Aniello Mennella
Simone Paradiso

Sara Bertocco
Samuele Galeotta
Gianmarco Maggio
Michele Maris
Daniele Tavagnacco
Andrea Zacchei

HELSINGIN YLIOPISTO
HELSINGFORS UNIVERSITET
UNIVERSITY OF HELSINKI

Elina Keihänen
Anna-Stiina Suur-Uski

Stelios Bollanos
Stratos Gerakakis
Maria Ieoronymaki
Ilias Ioannou

External collaborators

Brandon Hensley

Jeff Jewell

Reijo Keskitalo

Bruce Partridge

Martin Reinecke

Beyond PLANCK

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- "*BeyondPlanck*"
  - COMPET-4 program
  - PI: Hans Kristian Eriksen
  - Grant no.: 776282
  - Period: Mar 2018 to Nov 2020

Collaborating projects:

- "*bits2cosmology*"
  - ERC Consolidator Grant
  - PI: Hans Kristian Eriksen
  - Grant no: 772 253
  - Period: April 2018 to March 2023
- "*Cosmoglobe*"
  - ERC Consolidator Grant
  - PI: Ingunn Wehus
  - Grant no: 819 478
  - Period: June 2019 to May 2024

European Commission

Questions?

Beyond PLANCK

Commander

HELSINGIN YLIOPISTO
HELSINGFORS UNIVERSITET
UNIVERSITY OF HELSINKI

planetek hellas

JPL

BERKELEY LAB

HAVERFORD COLLEGE

THE FRAMEWORK PROGRAMME FOR RESEARCH AND INNOVATION
HORIZON 2020

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Cosmoglobe

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Backups

Low-ℓ likelihood – why KL compression?