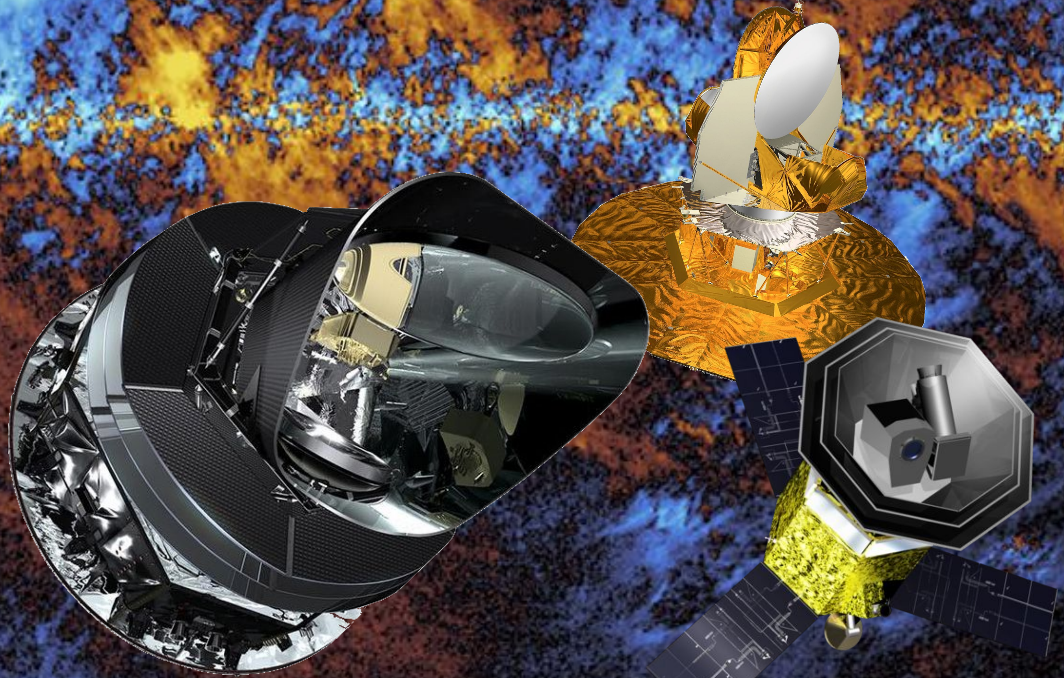
Intensity foregrounds and priors

Kristian Joten Andersen



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The BeyondPlanck data model

$$d_{j,t} = g_{j,t}\mathsf{P}_{tp,j}\left[\mathsf{B}^{\mathrm{symm}}_{pp',j}\sum_{c}\mathsf{M}_{cj}(\beta_{p'},\Delta^{j}_{\mathrm{bp}})a^{c}_{p'} + \mathsf{B}^{\mathrm{asymm}}_{j,t}\left(\boldsymbol{s}^{\mathrm{orb}}_{j} + \boldsymbol{s}^{\mathrm{fsl}}_{t}\right)\right] + n^{\mathrm{corr}}_{j,t} + n^{\mathrm{w}}_{j,t}.$$

$$\begin{aligned}
\boldsymbol{g} &\leftarrow P(\boldsymbol{g} \mid \boldsymbol{d}, \xi_n, \Delta_{\mathrm{bp}}, \boldsymbol{a}, \beta, C_\ell)\\
\boldsymbol{n}_{\mathrm{corr}} &\leftarrow P(\boldsymbol{n}_{\mathrm{corr}} \mid \boldsymbol{d}, \boldsymbol{g}, \xi_n, \Delta_{\mathrm{bp}}, \boldsymbol{a}, \beta, C_\ell)\\
\xi_n &\leftarrow P(\xi_n \mid \boldsymbol{d}, \boldsymbol{g}, \boldsymbol{n}_{\mathrm{corr}}, \Delta_{\mathrm{bp}}, \boldsymbol{a}, \beta, C_\ell)\\
\Delta_{\mathrm{bp}} &\leftarrow P(\Delta_{\mathrm{bp}} \mid \boldsymbol{d}, \boldsymbol{g}, \boldsymbol{n}_{\mathrm{corr}}, \xi_n, \boldsymbol{a}, \beta, C_\ell)\\
\beta &\leftarrow P(\beta \mid \boldsymbol{d}, \boldsymbol{g}, \boldsymbol{n}_{\mathrm{corr}}, \xi_n, \Delta_{\mathrm{bp}}, C_\ell)\\
\boldsymbol{a} &\leftarrow P(\boldsymbol{a} \mid \boldsymbol{d}, \boldsymbol{g}, \boldsymbol{n}_{\mathrm{corr}}, \xi_n, \Delta_{\mathrm{bp}}, \beta, C_\ell)\\
C_\ell &\leftarrow P(C_\ell \mid \boldsymbol{d}, \boldsymbol{g}, \boldsymbol{n}_{\mathrm{corr}}, \xi_n, \Delta_{\mathrm{bp}}, \boldsymbol{a}, \beta)
\end{aligned}$$

The posterior distribution

$$P(\mathbf{a}, \beta \,|\, \mathbf{d}, \mathbf{g}, ...) = P(\mathbf{a}, \beta \,|\, \mathbf{d})$$

for simpler notation

Bayes' theorem:

$$P(\mathbf{a}, \beta \,|\, \mathbf{d}) = \frac{P(\mathbf{d} \,|\, \mathbf{a}, \beta) P(\mathbf{a}, \beta)}{P(\mathbf{d})} \propto \mathcal{L}(\mathbf{a}, \beta) P(\mathbf{a}, \beta)$$

Likelihood function

Priors

We'll get to these later

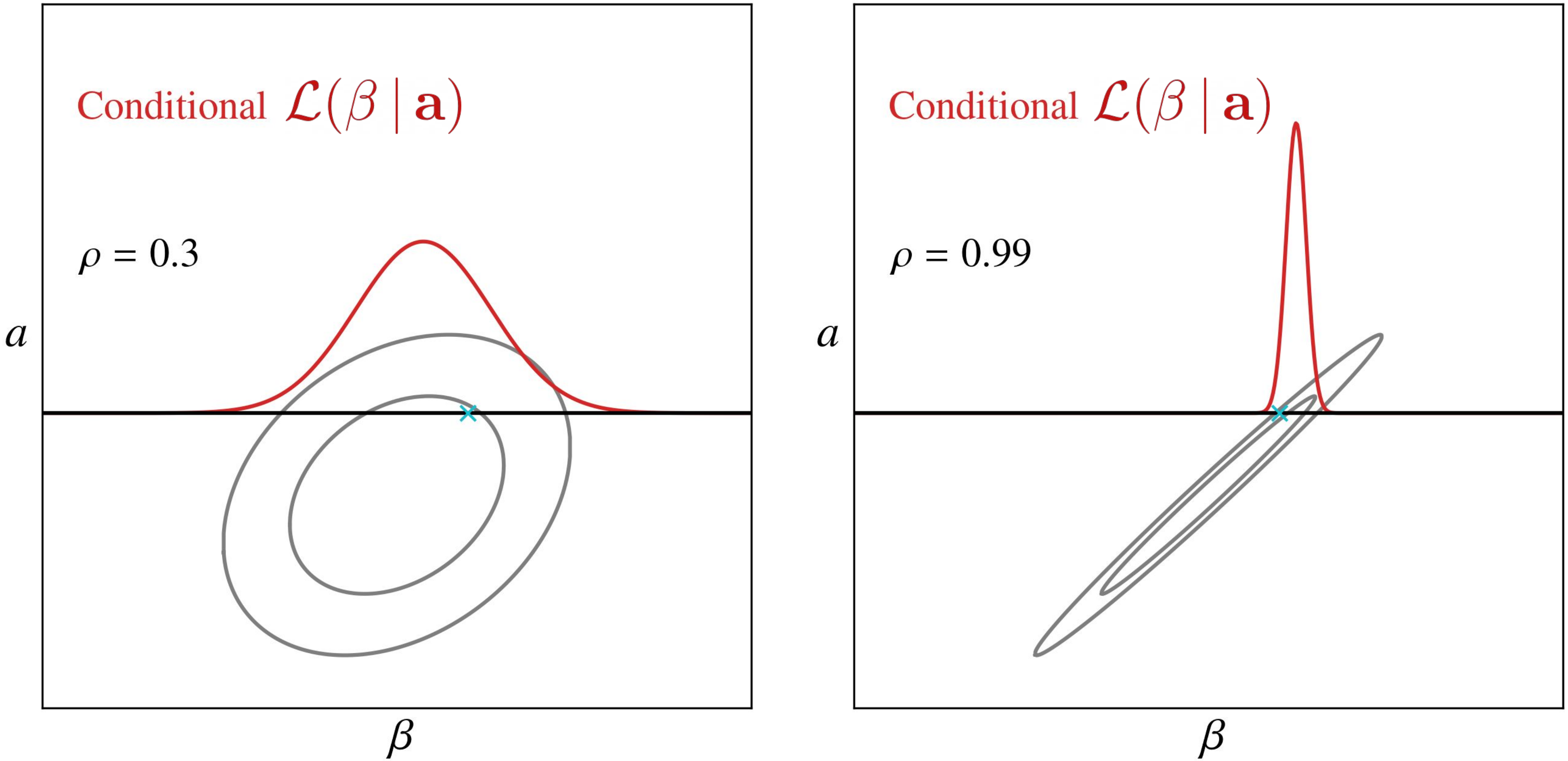
Gibbs sampling:

$$\beta \leftarrow P(\beta \,|\, \mathbf{d}, \mathbf{a}, \ldots) \propto \mathcal{L}(\beta \,|\, \mathbf{a}) P(\beta \,|\, \mathbf{a})$$

$$\mathbf{a} \leftarrow P(\mathbf{a} \,|\, \mathbf{d}, \beta, \ldots) \propto \mathcal{L}(\mathbf{a} \,|\, \beta) P(\mathbf{a} \,|\, \beta)$$

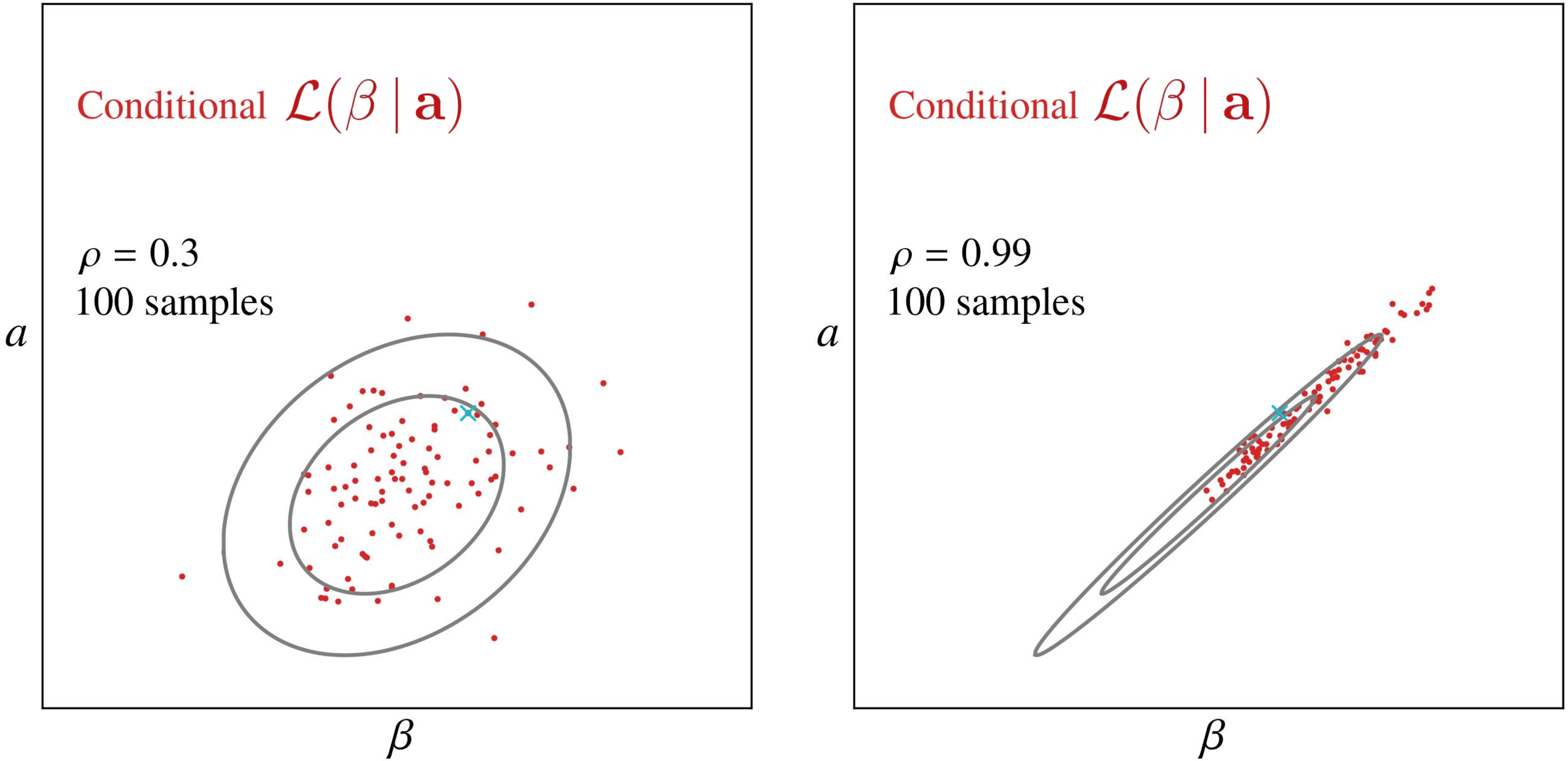
The posterior distribution

$$P(\mathbf{a}, \beta \,|\, \mathbf{d}) = \frac{P(\mathbf{d} \,|\, \mathbf{a}, \beta) P(\mathbf{a}, \beta)}{P(\mathbf{d})} \propto \mathcal{L}(\mathbf{a}, \beta) P(\mathbf{a}, \beta)$$



The posterior distribution

$$P(\mathbf{a}, \beta \,|\, \mathbf{d}) = \frac{P(\mathbf{d} \,|\, \mathbf{a}, \beta) P(\mathbf{a}, \beta)}{P(\mathbf{d})} \propto \mathcal{L}(\mathbf{a}, \beta) P(\mathbf{a}, \beta)$$



The posterior distribution

$$P(\mathbf{a}, \beta \,|\, \mathbf{d}) = \frac{P(\mathbf{d} \,|\, \mathbf{a}, \beta) P(\mathbf{a}, \beta)}{P(\mathbf{d})} \propto \mathcal{L}(\mathbf{a}, \beta) P(\mathbf{a}, \beta)$$

Likelihood function

$$\mathcal{L}(\mathbf{a}, \beta) = \mathcal{L}(\mathbf{a} \,|\, \beta) \mathcal{L}(\beta)$$

Joint

Conditional

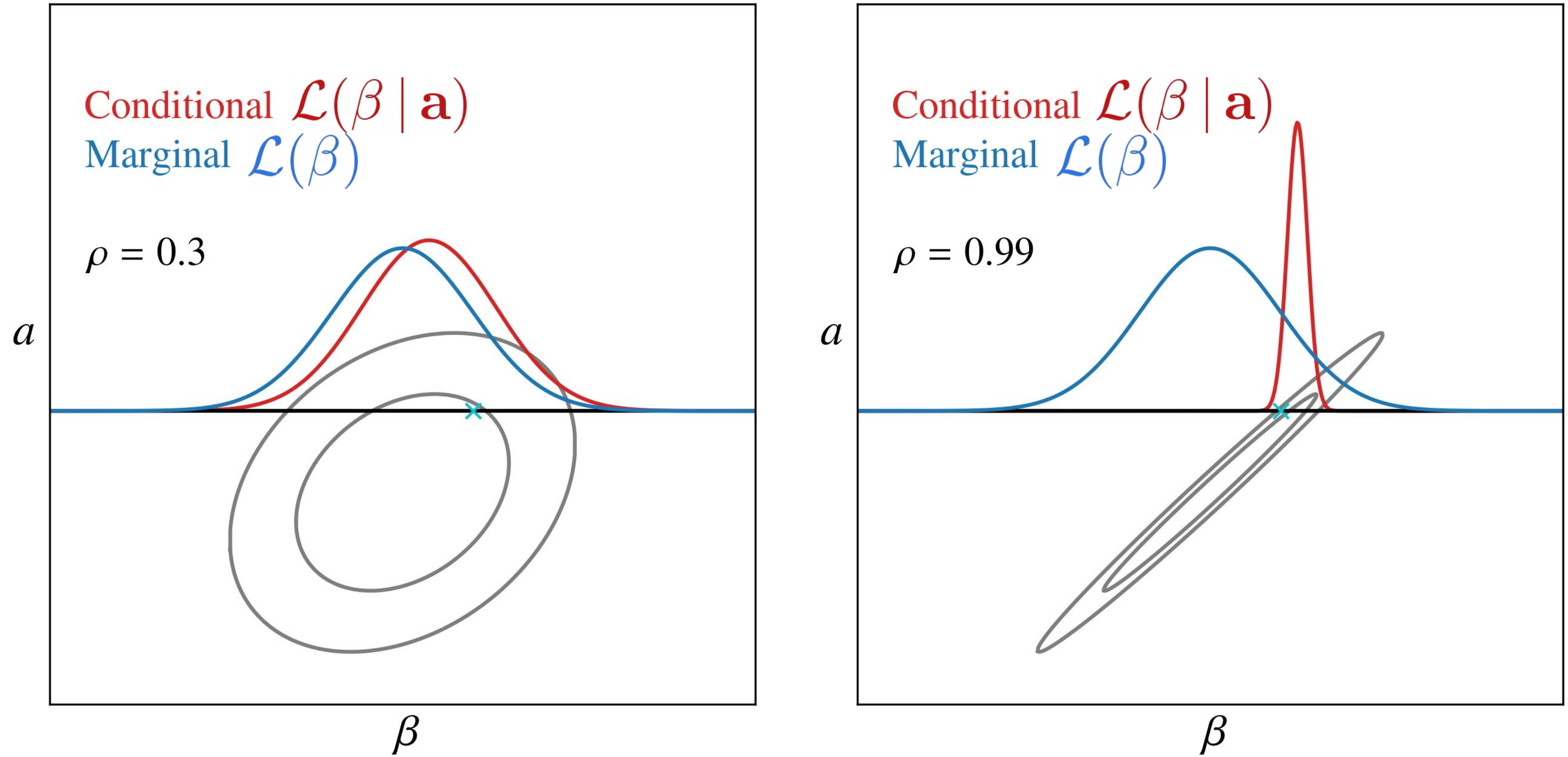
Marginal

$$\beta \leftarrow \mathcal{L}(\beta)$$

$$\mathbf{a} \leftarrow \mathcal{L}(\mathbf{a} \,|\, \beta)$$

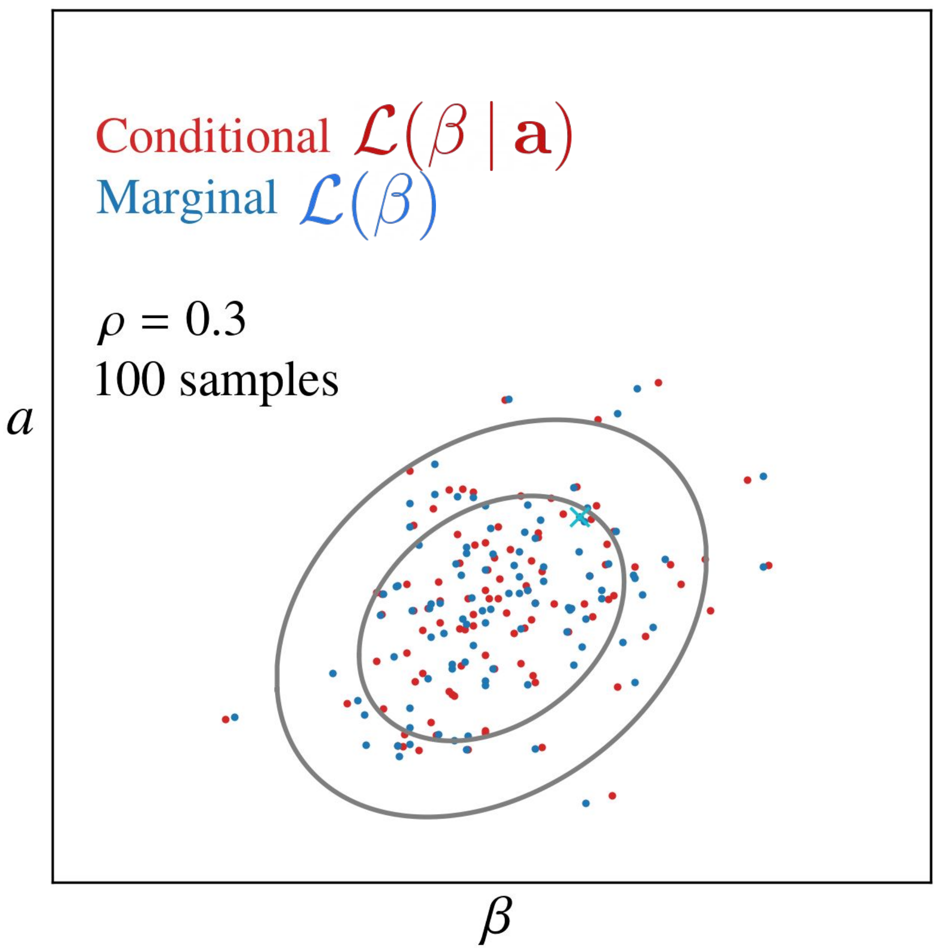
The posterior distribution

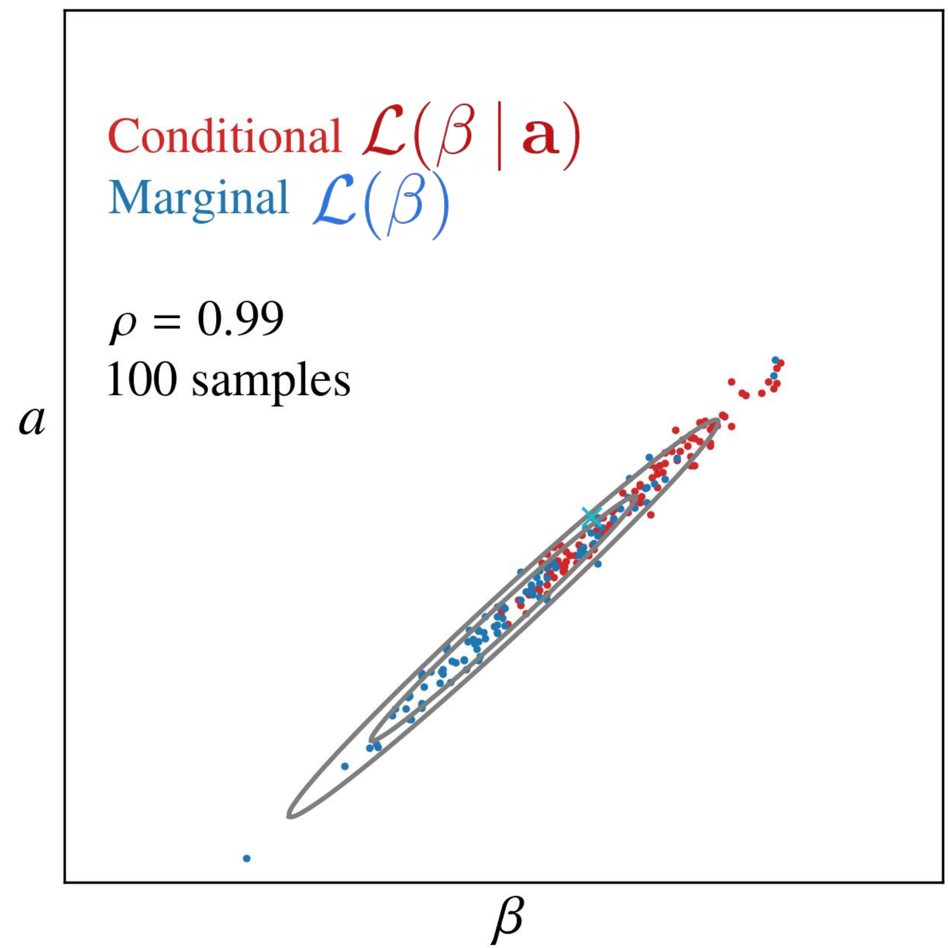
$$P(\mathbf{a}, \beta \,|\, \mathbf{d}) = \frac{P(\mathbf{d} \,|\, \mathbf{a}, \beta) P(\mathbf{a}, \beta)}{P(\mathbf{d})} \propto \mathcal{L}(\mathbf{a}, \beta) P(\mathbf{a}, \beta)$$



The posterior distribution

$$P(\mathbf{a}, \beta \,|\, \mathbf{d}) = \frac{P(\mathbf{d} \,|\, \mathbf{a}, \beta) P(\mathbf{a}, \beta)}{P(\mathbf{d})} \propto \mathcal{L}(\mathbf{a}, \beta) P(\mathbf{a}, \beta)$$





Conditional vs. marginal likelihood function

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$$\mathcal{L}(\boldsymbol{a},\beta) \propto \frac{e^{-\frac{1}{2}(\boldsymbol{m}_\nu - \mathrm{A}_\nu \boldsymbol{a})^T \mathrm{N}_\nu^{-1} (\boldsymbol{m}_\nu - \mathrm{A}_\nu \boldsymbol{a})}}{\sqrt{|2\pi \mathrm{N}_\nu^{-1}|}}$$

assuming gaussian white noise

$$-2\ln\mathcal{L}(\boldsymbol{a},\beta) = \mathrm{const} + (\boldsymbol{m}_\nu - \mathrm{A}_\nu \boldsymbol{a})^T \mathrm{N}_\nu^{-1} (\boldsymbol{m}_\nu - \mathrm{A}_\nu \boldsymbol{a}) = \mathrm{const} + \chi^2$$

Assume for computational efficiency:

- β varies more slowly than ***a***
- uniform pixelization and angular resolution for all $\boldsymbol{m}_\nu$
- N_ν is diagonal

Stompor et al. (2009, MNRAS, 392, 216) shows that per pixel (across all frequencies) the log-likelihood becomes

$$-2\ln\mathcal{L}_{\mathrm{ridge}}(\beta) = \mathrm{const} + (\mathrm{A}^T \mathrm{N}^{-1} \boldsymbol{m})^T (\mathrm{A}^T \mathrm{N}^{-1} \mathrm{A})^{-1} (\mathrm{A}^T \mathrm{N}^{-1} \boldsymbol{m}),$$

$$-2\ln\mathcal{L}_{\mathrm{marg}}(\beta) = -2\ln \int d\boldsymbol{a}\, \exp\left[-\frac{1}{2}(\boldsymbol{m} - \mathrm{A}\boldsymbol{a})^T \mathrm{N}^{-1} (\boldsymbol{m} - \mathrm{A}\boldsymbol{a})\right]$$

$$= \mathrm{const} + (\mathrm{A}^T \mathrm{N}^{-1} \boldsymbol{m})^T (\mathrm{A}^T \mathrm{N}^{-1} \mathrm{A})^{-1} (\mathrm{A}^T \mathrm{N}^{-1} \boldsymbol{m})$$

$$+ \ln\left|(\mathrm{A}^T \mathrm{N}^{-1} \mathrm{A})^{-1}\right|,$$

Beyond PLANCK

Data model and priors

Data model and external data

Free parameters

$$
\begin{aligned}
s_{\mathrm{RJ}} = & \left(a_{\mathrm{CMB}} + a_{\mathrm{quad}}(\nu)\right) \frac{x^2 e^x}{(e^x - 1)^2} + \\
& + a_{\mathrm{s}} \left(\frac{\nu}{\nu_{0,\mathrm{s}}}\right)^{\beta_{\mathrm{s}}} + \\
& + a_{\mathrm{ff}} \left(\frac{\nu_{0,\mathrm{ff}}}{\nu}\right)^2 \frac{g_{\mathrm{ff}}(\nu; T_{\mathrm{e}})}{g_{\mathrm{ff}}(\nu_{0,\mathrm{ff}}; T_{\mathrm{e}})} + \\
& + a_{\mathrm{sd}} \left(\frac{\nu_{0,\mathrm{sd}}}{\nu}\right)^2 \frac{f_{\mathrm{sd}}\left(\nu \cdot \frac{\nu_{\mathrm{p}}}{30.0\,\mathrm{GHz}}\right)}{f_{\mathrm{sd}}\left(\nu_{0,\mathrm{sd}} \cdot \frac{\nu_{\mathrm{p}}}{30.0\,\mathrm{GHz}}\right)} + \\
& + a_{\mathrm{d}} \left(\frac{\nu}{\nu_{0,\mathrm{d}}}\right)^{\beta_{\mathrm{d}}+1} \frac{e^{h\nu_{0,\mathrm{d}}/k_{\mathrm{B}}T_{\mathrm{d}}} - 1}{e^{h\nu/k_{\mathrm{B}}T_{\mathrm{d}}} - 1} + \\
& + \sum_{j=1}^{N_{\mathrm{src}}} a_{j,\mathrm{src}} \left(\frac{\nu}{\nu_{0,\mathrm{src}}}\right)^{\alpha_{j,\mathrm{src}}-2},
\end{aligned}
$$

CMB

Synchrotron

Free-free

AME / Spinning dust

Thermal dust

Point sources

Not: Sunyaev-Zeldovich effect, zodiacal light, cosmic infrared background

Data model and external data

Free parameters

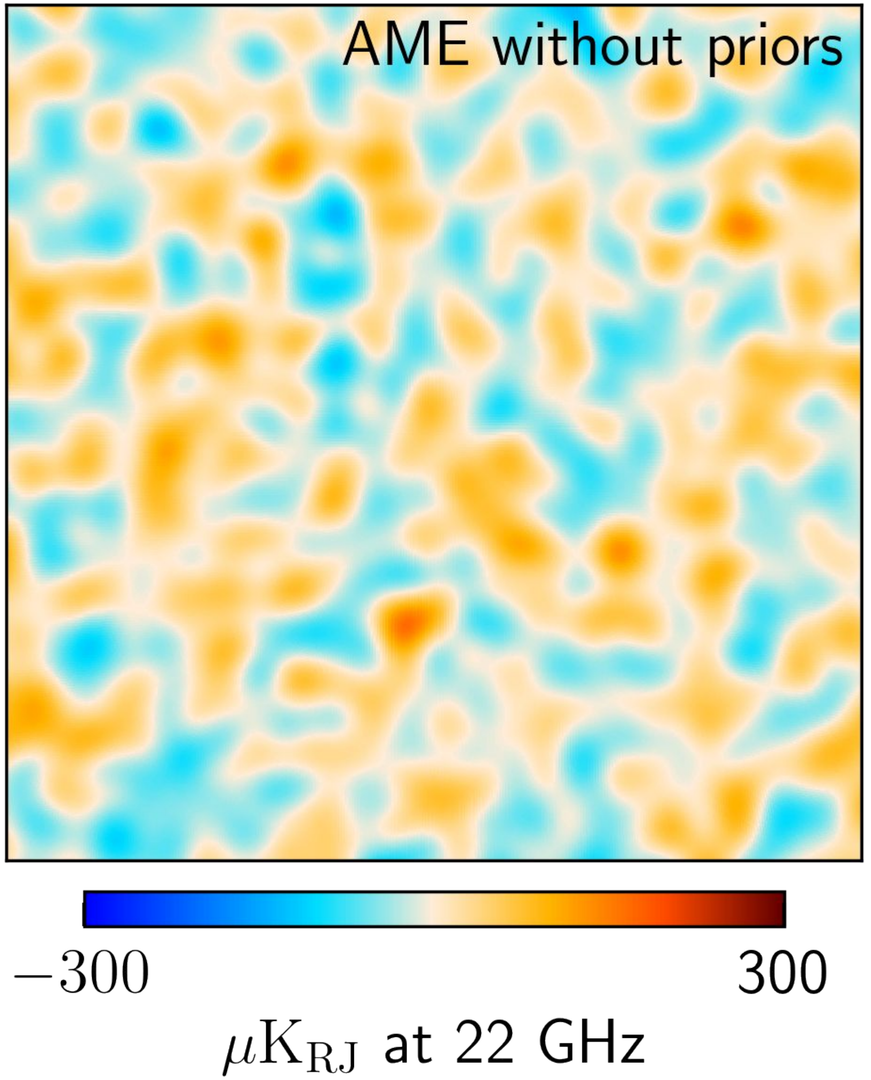
$$
\begin{aligned}
s_{\mathrm{RJ}} = {} & \left(a_{\mathrm{CMB}} + a_{\mathrm{quad}}(\nu)\right)\frac{x^2 e^x}{(e^x-1)^2} + \\
& + a_{\mathrm{s}}\left(\frac{\nu}{\nu_{0,\mathrm{s}}}\right)^{\beta_{\mathrm{s}}} + \\
& + a_{\mathrm{ff}}\left(\frac{\nu_{0,\mathrm{ff}}}{\nu}\right)^{2}\frac{g_{\mathrm{ff}}(\nu;T_{\mathrm{e}})}{g_{\mathrm{ff}}(\nu_{0,\mathrm{ff}};T_{\mathrm{e}})} + \\
& + a_{\mathrm{sd}}\left(\frac{\nu_{0,\mathrm{sd}}}{\nu}\right)^{2}\frac{f_{\mathrm{sd}}\left(\nu\cdot\frac{\nu_{\mathrm{p}}}{30.0\,\mathrm{GHz}}\right)}{f_{\mathrm{sd}}\left(\nu_{0,\mathrm{sd}}\cdot\frac{\nu_{\mathrm{p}}}{30.0\,\mathrm{GHz}}\right)} + \\
& + a_{\mathrm{d}}\left(\frac{\nu}{\nu_{0,\mathrm{d}}}\right)^{\beta_{\mathrm{d}}+1}\frac{e^{h\nu_{0,\mathrm{d}}/k_{\mathrm{B}}T_{\mathrm{d}}}-1}{e^{h\nu/k_{\mathrm{B}}T_{\mathrm{d}}}-1} + \\
& + \sum_{j=1}^{N_{\mathrm{src}}} a_{j,\mathrm{src}}\left(\frac{\nu}{\nu_{0,\mathrm{src}}}\right)^{\alpha_{j,\mathrm{src}}-2},
\end{aligned}
$$

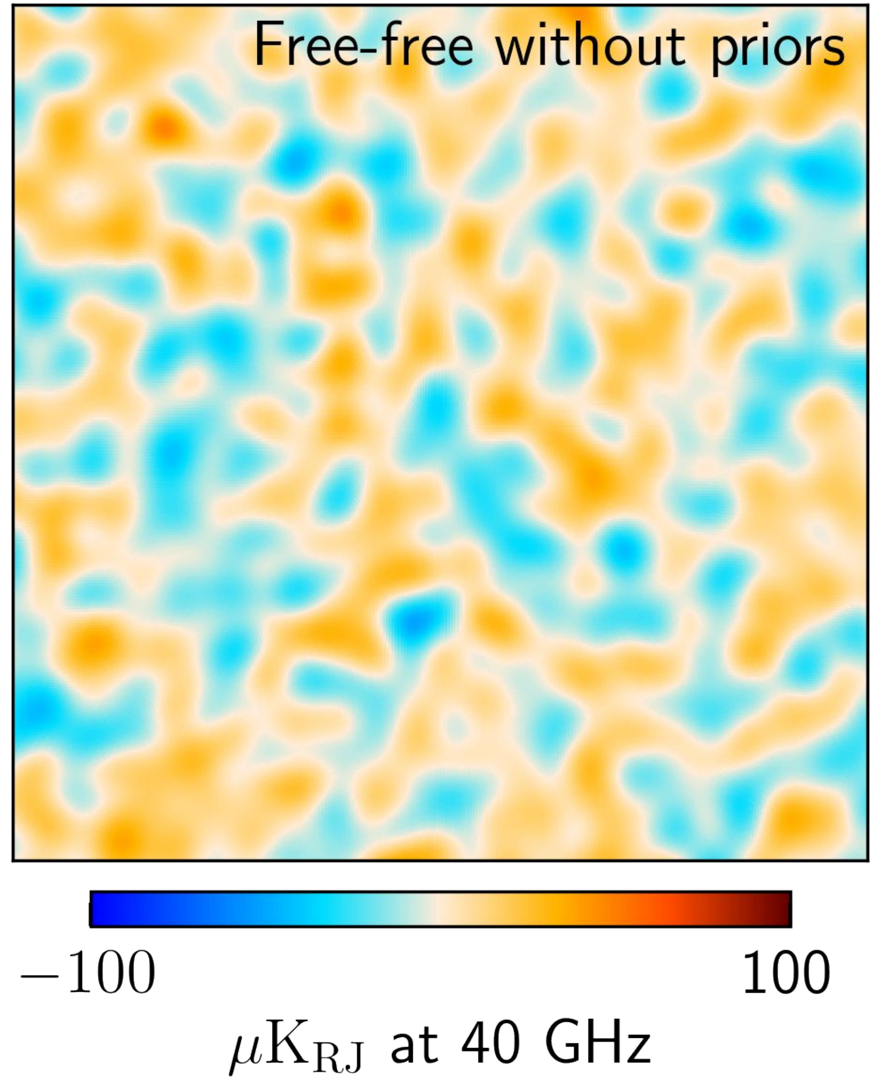
- Only the following data are included in the component separation analysis:
  - ***Planck LFI 30, 44 and 70 GHz pixel maps, binned from time-ordered data*** (Suur-Uski et al.2020)
  - ***Planck 857 GHz*** to constrain thermal dust intensity
  - ***WMAP 33-61 GHz*** to constrain low-frequency foregrounds
  - ***Haslam 408 MHz*** to constrain synchrotron intensity

- Intermediate *Planck HFI* and *WMAP 23* GHz data are ***not*** included, because they have similar or higher signal-to-noise ratios than *Planck* LFI, which we want to be the ***dominant*** statistical driver.

Amplitude degeneracies and priors

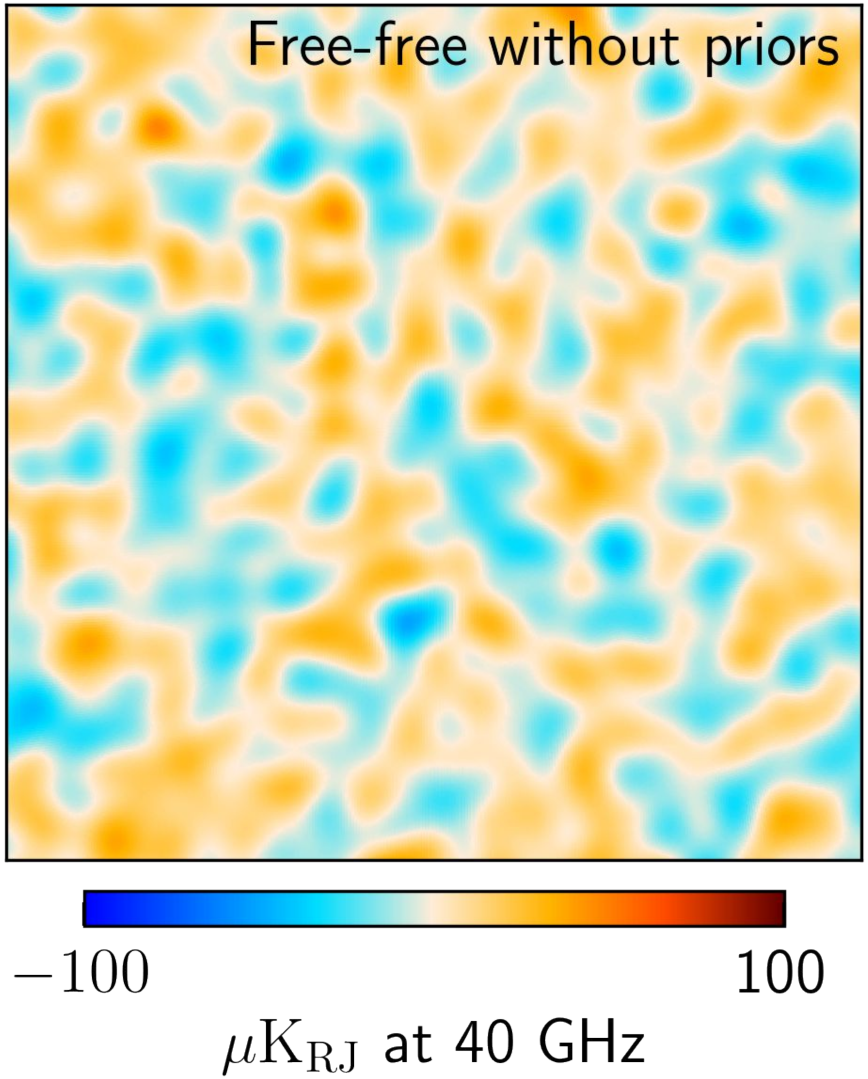
60' FWHM, $20^\circ \times 20^\circ$ centered on galactic south pole

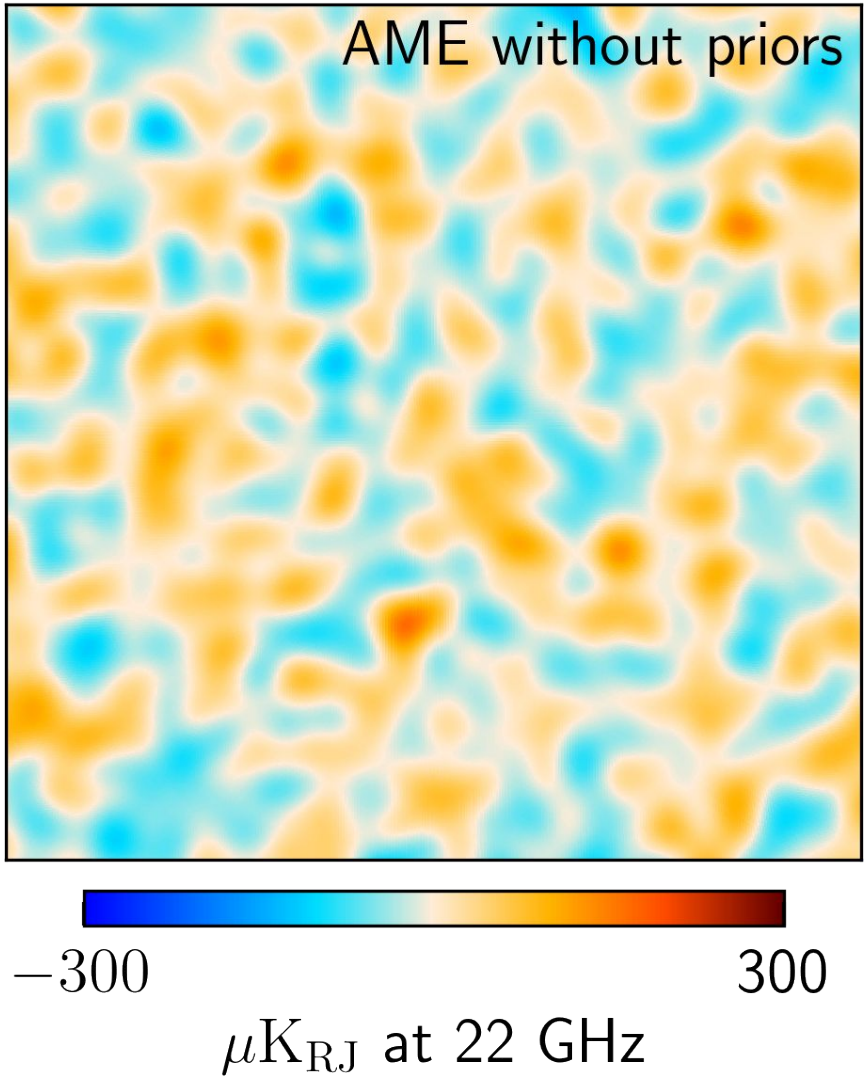




Amplitude degeneracies and priors

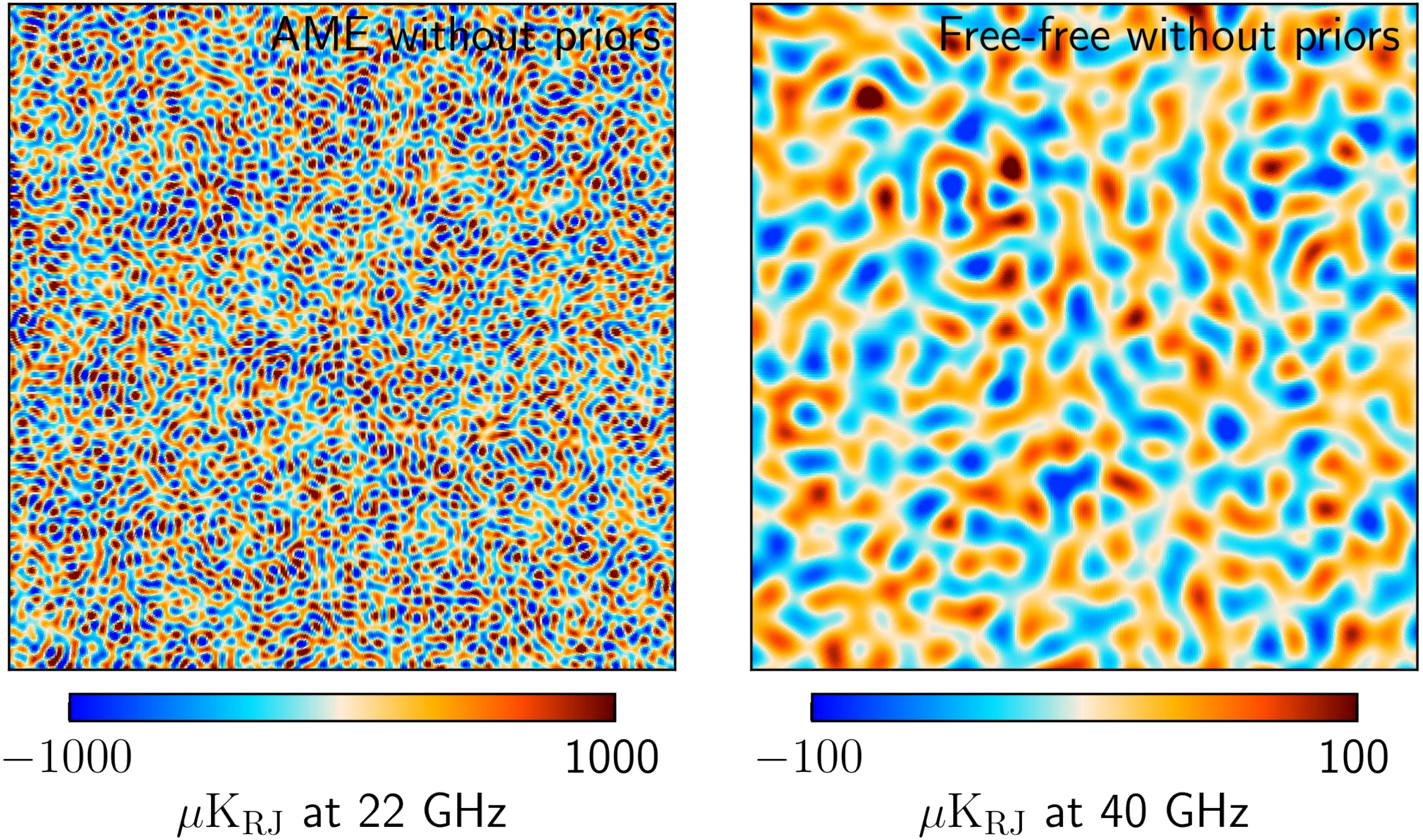
60' FWHM, $20^\circ \times 20^\circ$ centered on galactic south pole





Amplitude degeneracies and priors

15' FWHM, $20^\circ \times 20^\circ$ centered on galactic south pole



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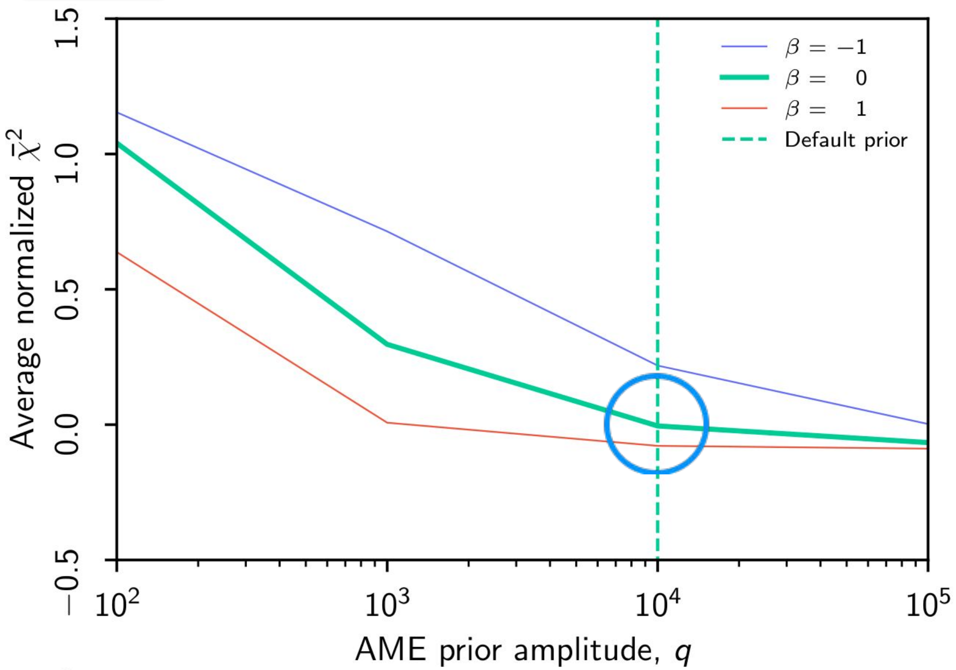
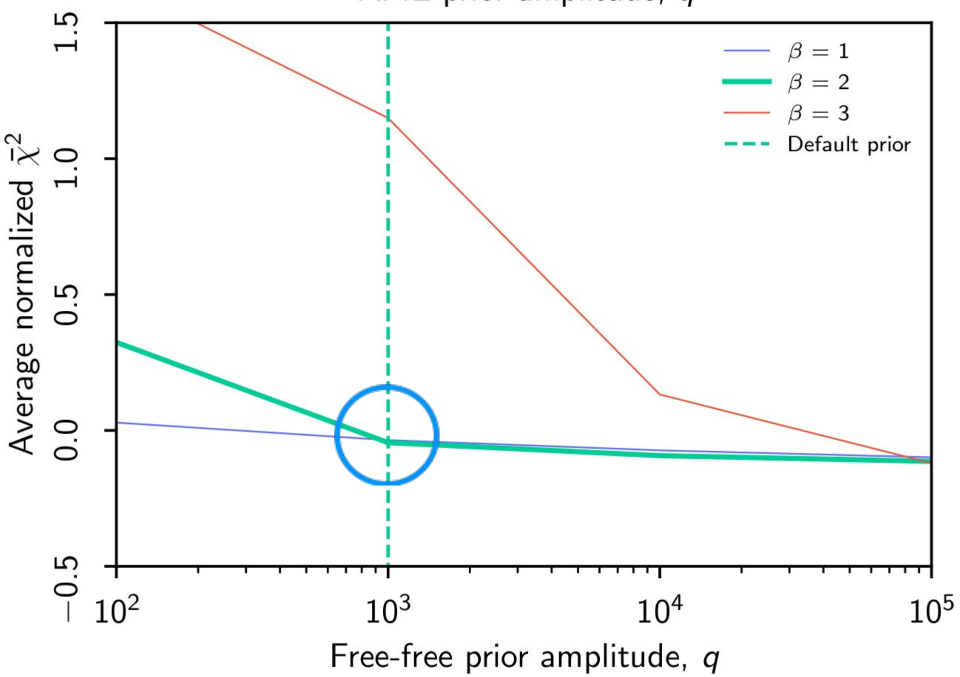
Amplitude degeneracies and priors





Implement an amplitude prior with mean maps based on:

- *Planck* 2015 free-free map,
- scaled *Planck* 857 GHz,

The prior RMS is specified as

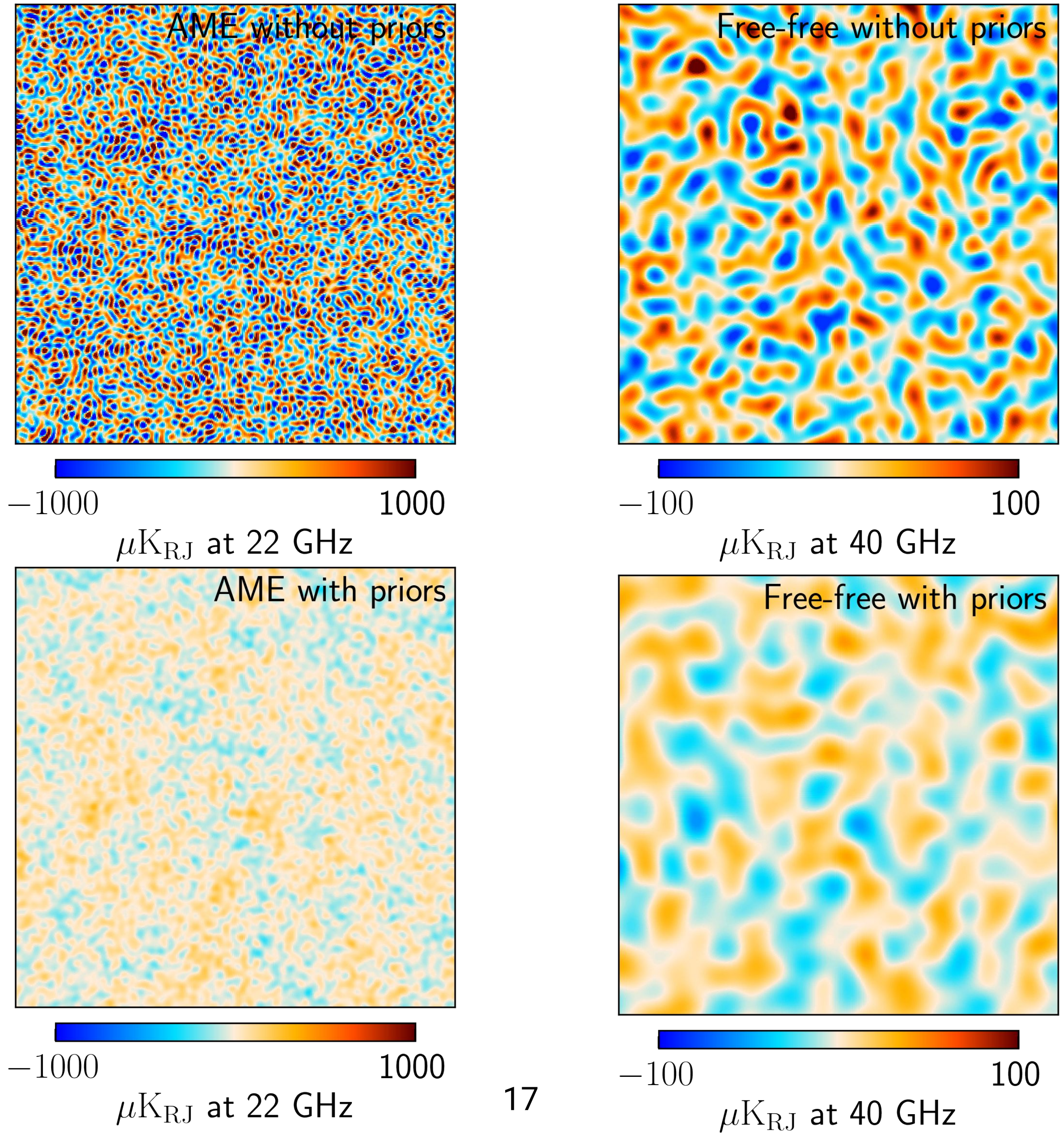
$$\hat{D}(\ell) = q\left(\frac{\ell}{\ell_0}\right)^{\beta}$$

q ~ prior amplitude
β ~ tilt parameter
ℓ_0 ~ pivot multipole = 50

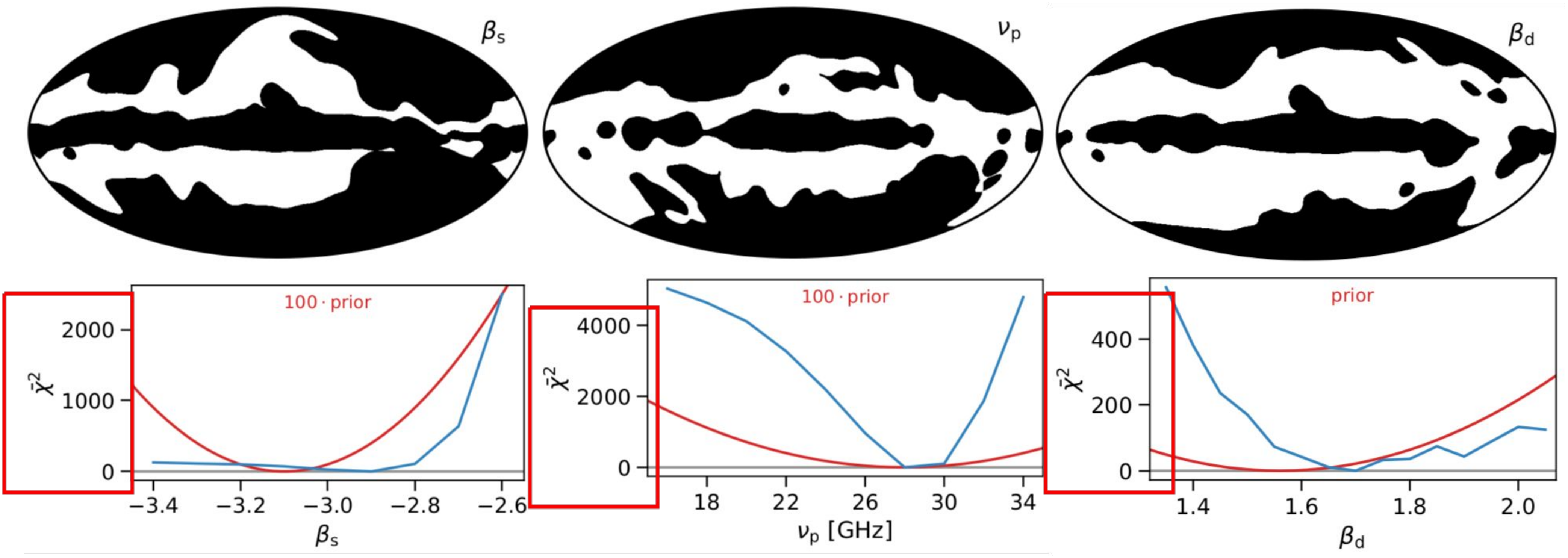
$$\bar{\chi}^2 = \frac{\chi^2 - \nu}{\sqrt{2\nu}}; \quad \nu = 15400$$

Amplitude degeneracies and priors

15’ FWHM, $20^\circ \times 20^\circ$ centered on galactic south pole



Spectral parameter degeneracies and priors



Well constrained by data.

Sample from data, with Gaussian prior $N(-3.1, 0.1^2)$. (*Planck* 2016, A&A, 594, A10)

Well constrained by data.

Sample from data, with Gaussian prior $N(28, 3^2)$ [GHz].

Prefers higher values, diverges to > 2.0 when sampled with other parameters

We only sample from Gaussian prior $N(1.56, 0.03^2)$. (*Planck* 2016, A&A, 594, A10)

Spectral parameter degeneracies and priors

$$\alpha_{\rm src} = N(-0.1, 0.3^2)$$

Bennet et al. 2013, ApJS, 208, 20

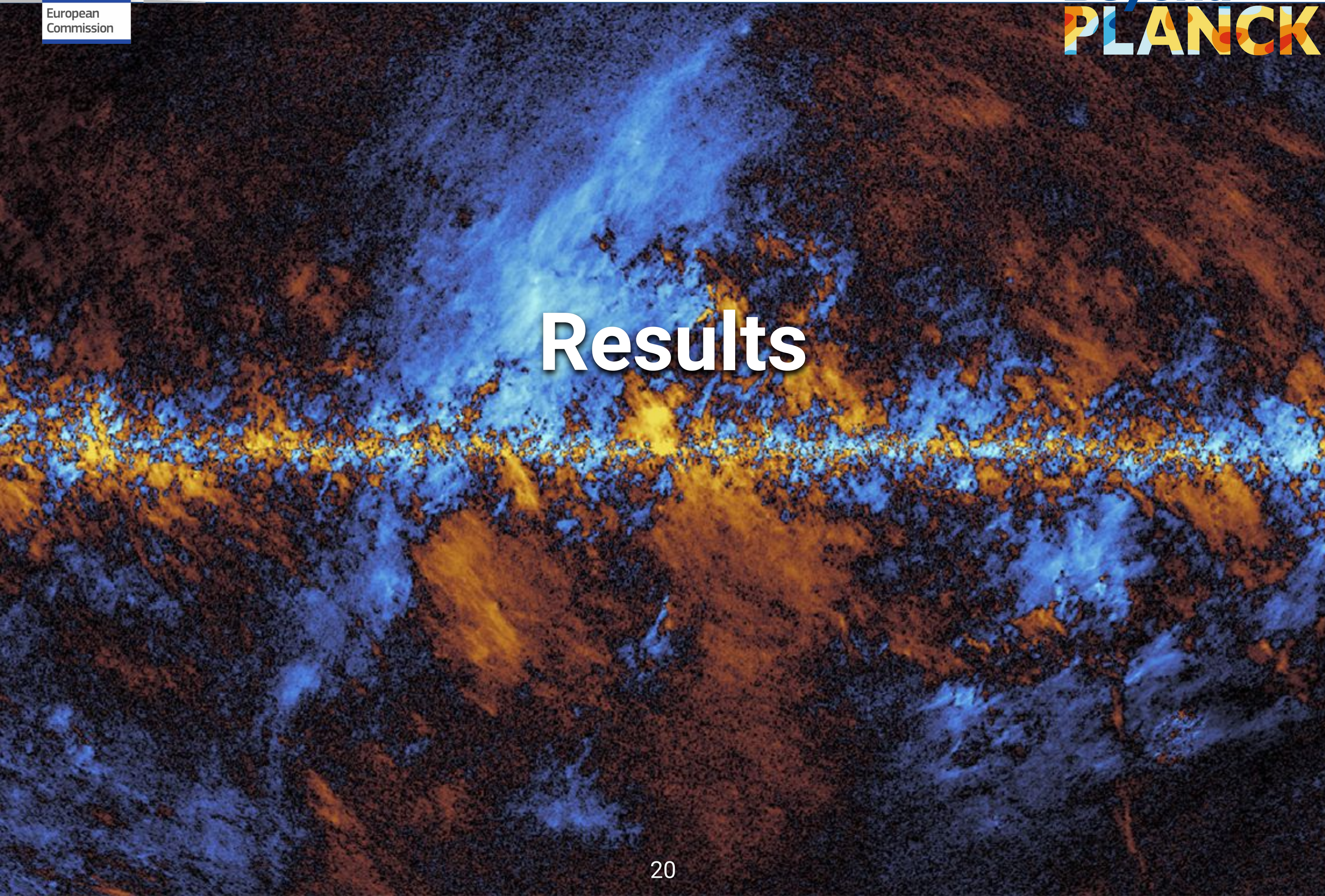
$$T_{\rm e} = 7000\,{\rm K}$$

Planck 2016, A&A, 594, A10

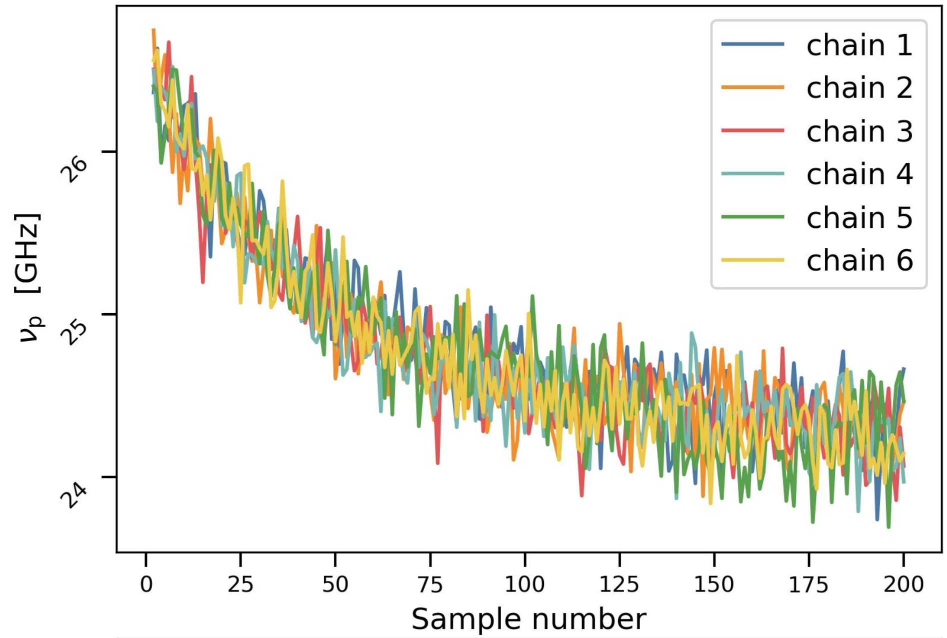
$$T_{\rm dust} = T_{\rm dust,HFI}$$

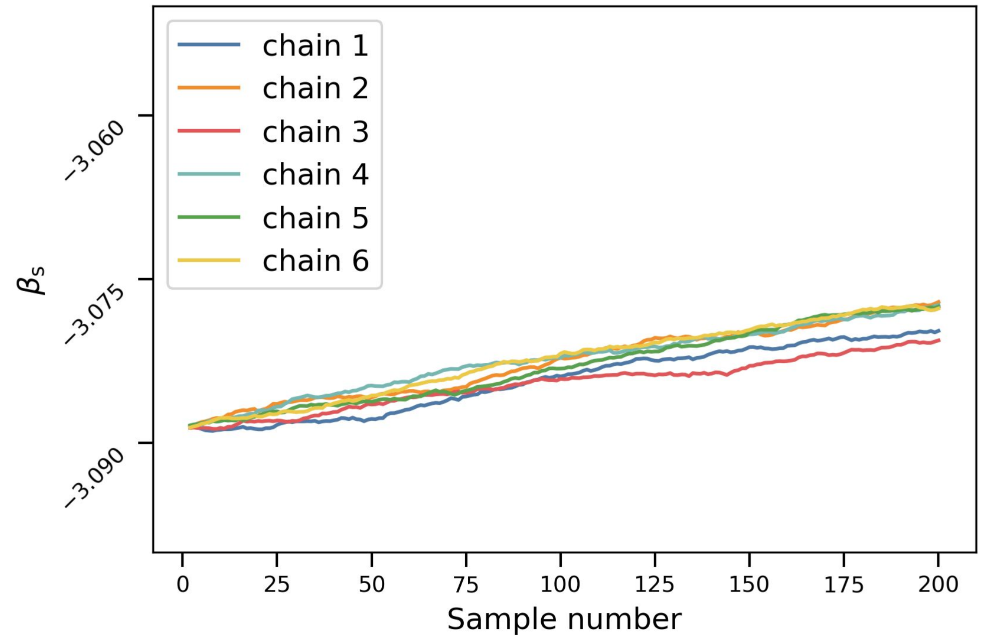
Planck Collaboration Int. LVII. 2020, A&A, in press [arXiv:2007.04997]
Most recent *Planck* HFI

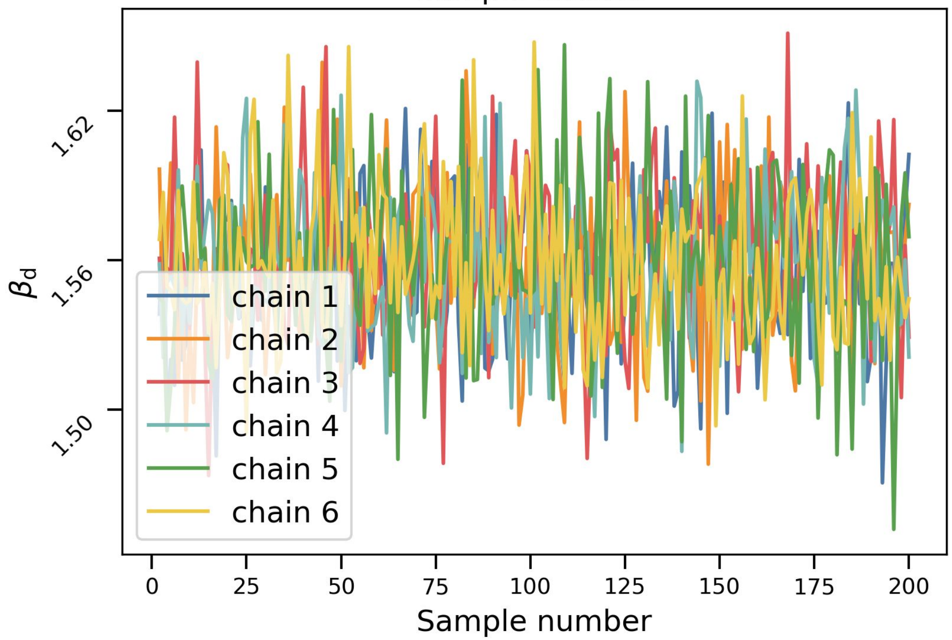
Results



Spectral parameters posteriors







Parameter	Value	Sample range
ν_p	24.31 ± 0.21 GHz	150 – 200
β_s	-3.0819 ± 0.0025	50 – 200
β_d	1.559 ± 0.031	50 – 200

Low frequency amplitudes

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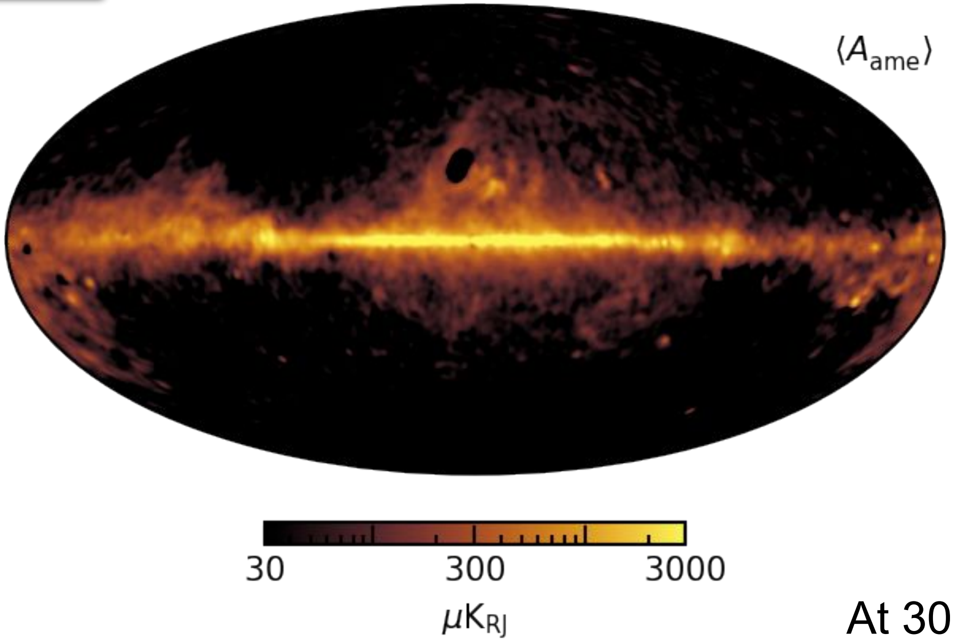
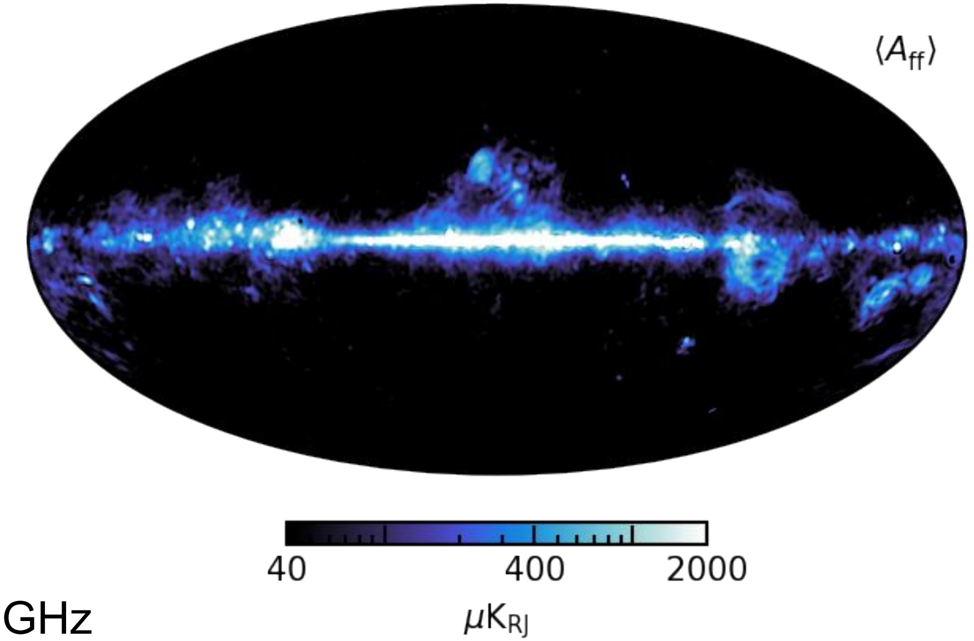
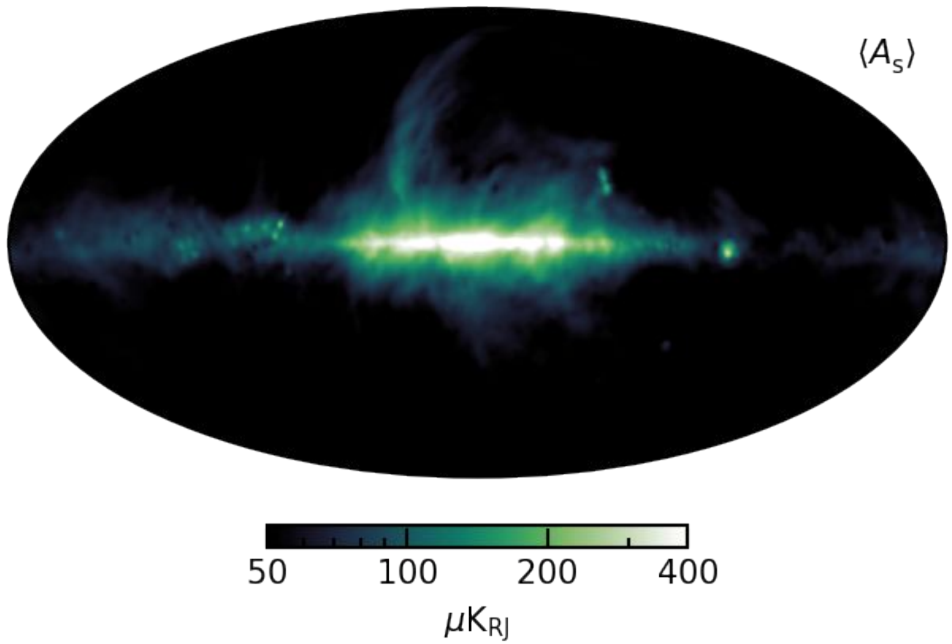
At 22 GHz



At 40 GHz



At 30 GHz



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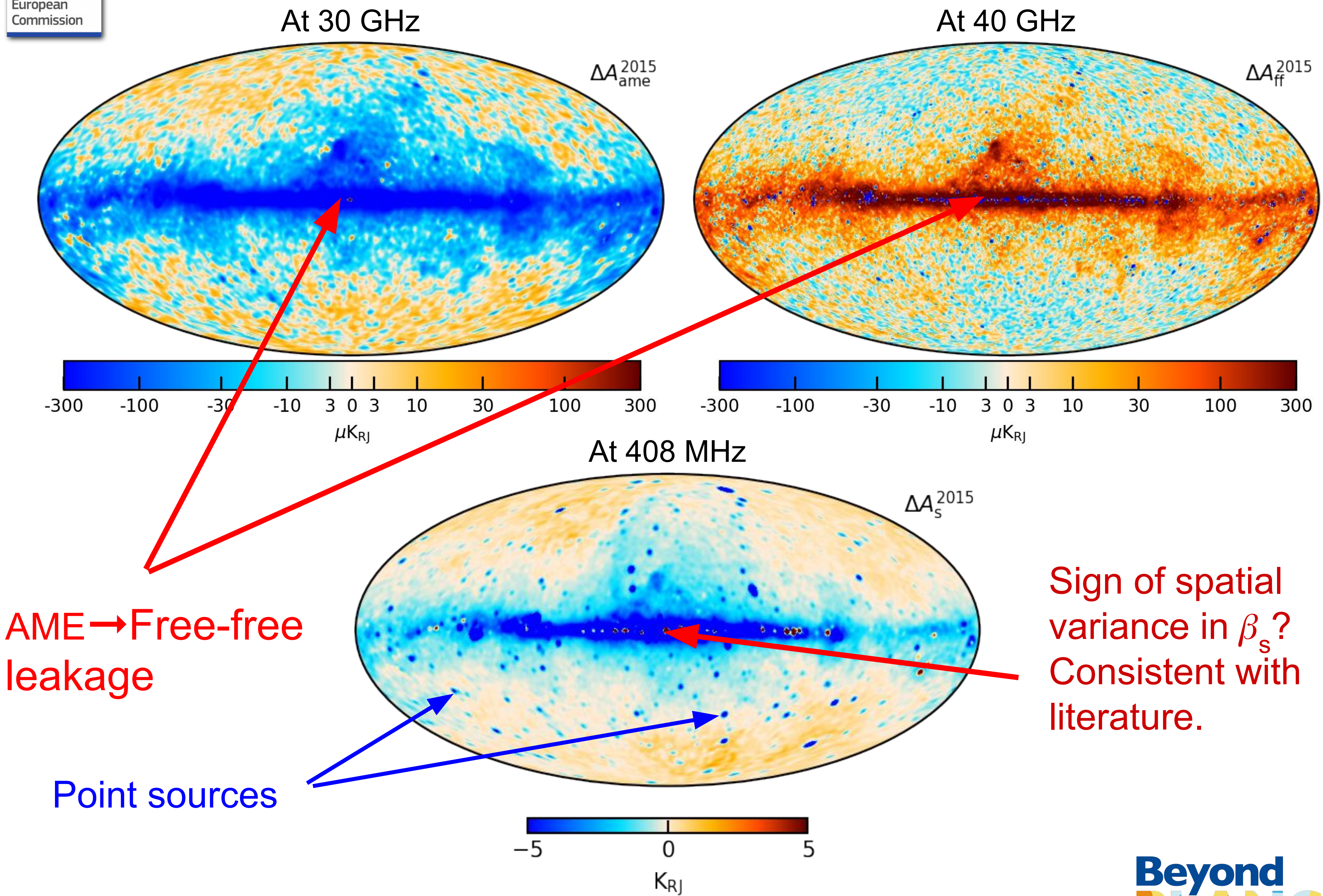
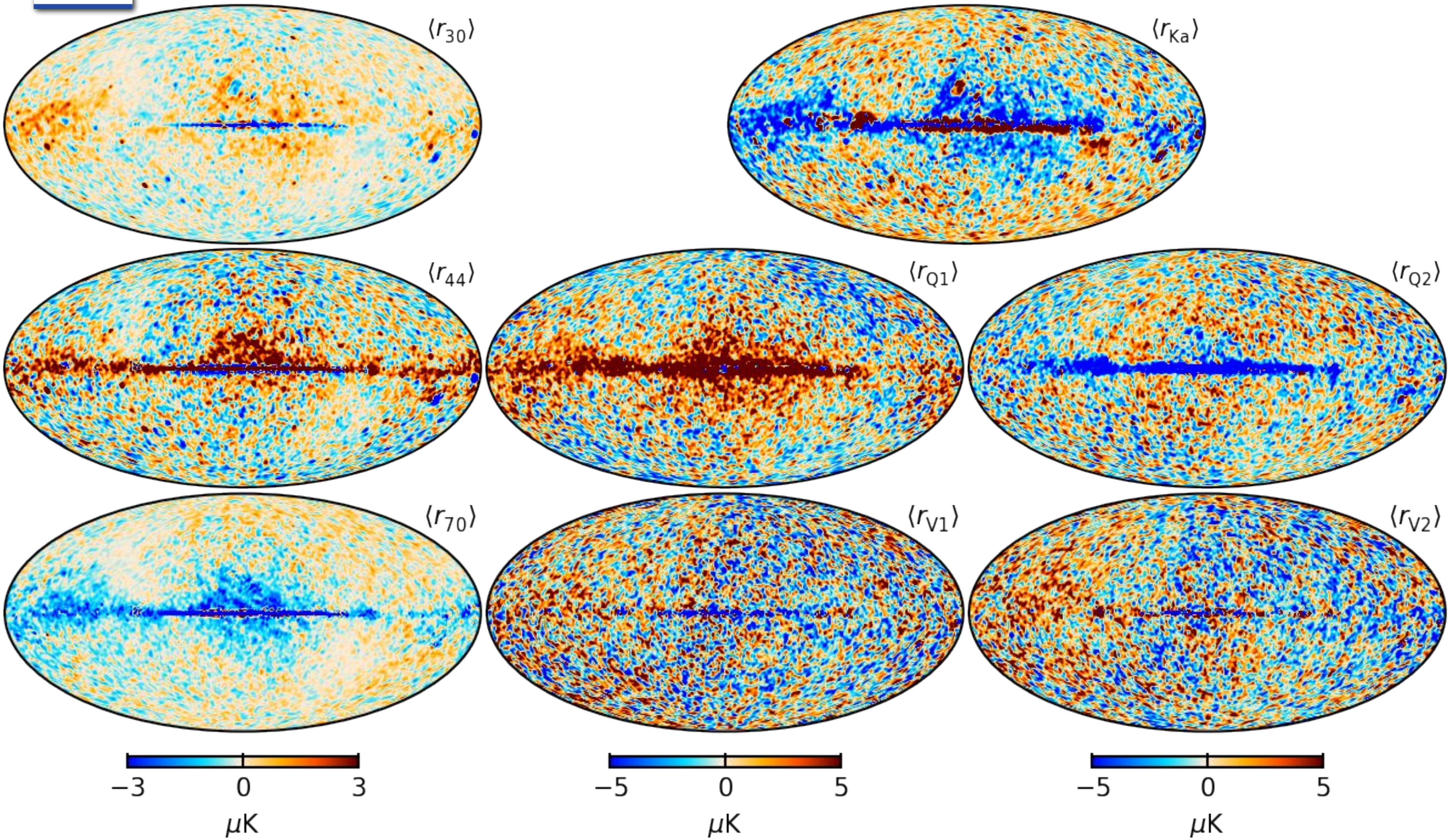
Low frequency amplitude difference with *Planck* 2015

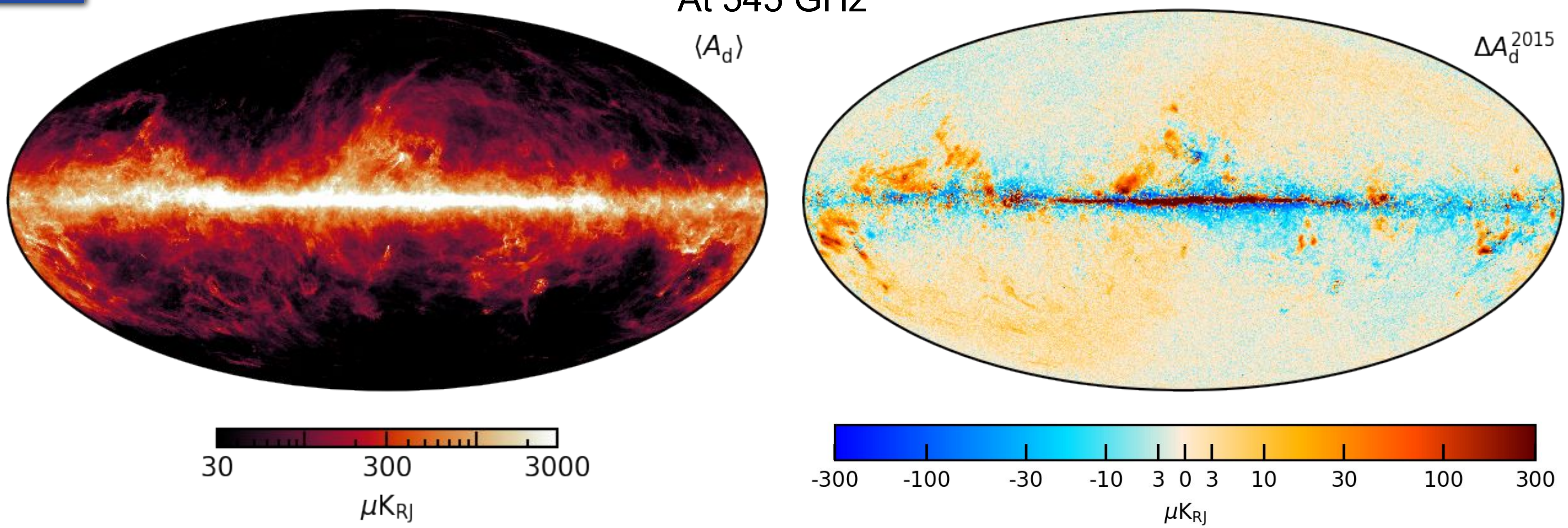


Goodness-of-fit: Residuals

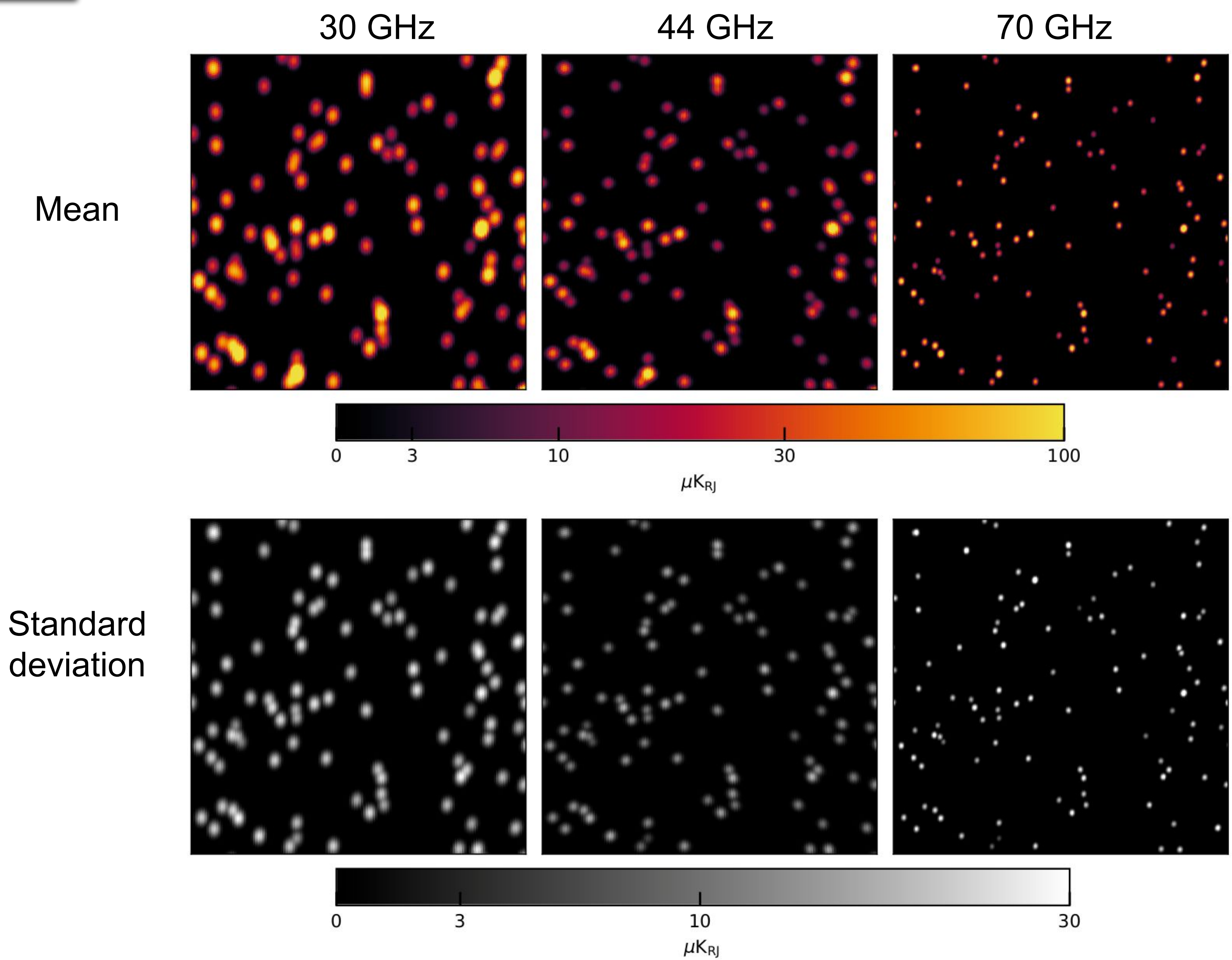


Thermal dust emission

At 545 GHz



Compact sources

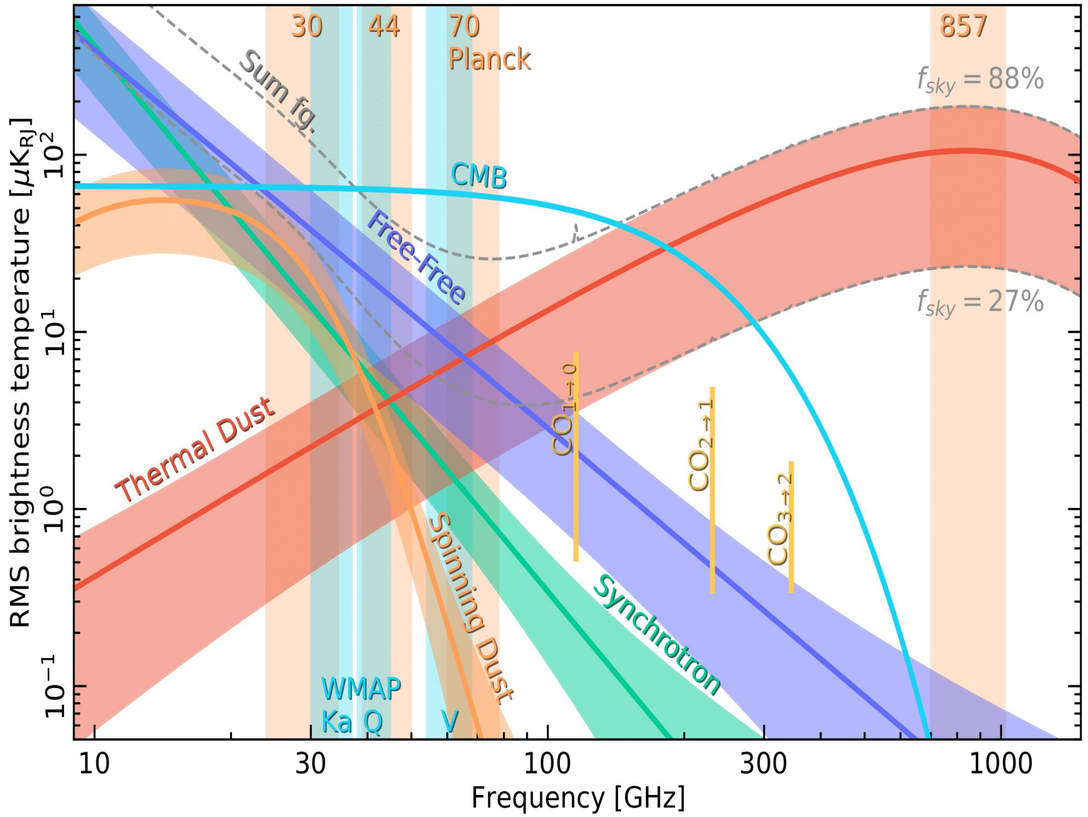


$20^\circ \times 20^\circ$
centered on
$(l, b) = (90^\circ, 70^\circ)$

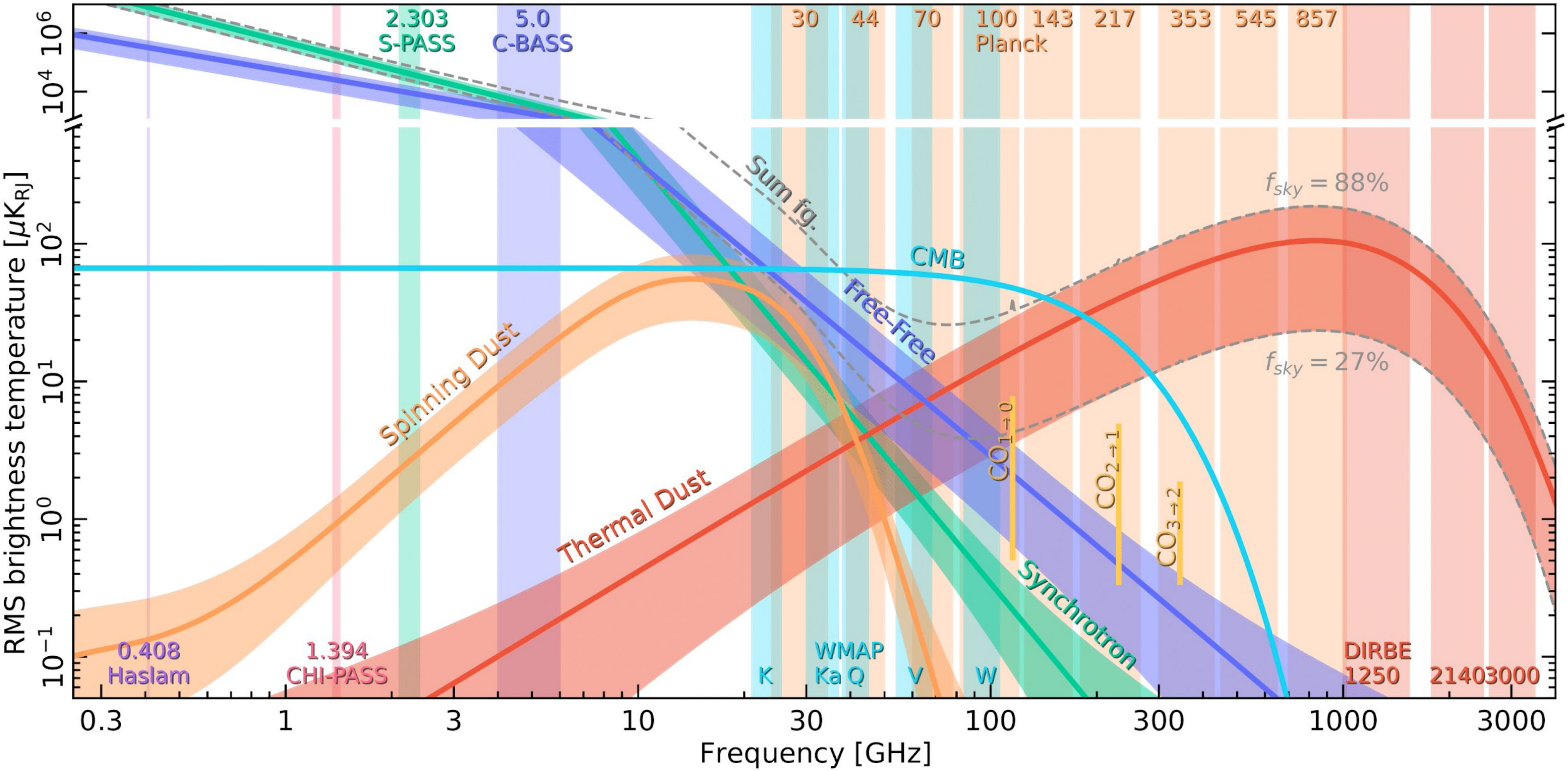
Outlook

Haslam
0.408

0.408 GHz



Outlook



The Cosmoglobe project

Summary

- Marginal likelihood ⇒ faster sampling of correlated posteriors
- Amplitude priors ⇒ reduce degeneracies; AME, free-free, CMB
- Spectral parameter priors ⇒ constrain weakly determined parameters while still propagating uncertainties

- Degeneracies between AME, free-free, and synchrotron amplitudes
- More datasets needed to break degeneracies.
- Long burn-in for AME and synchrotron, optimize pivot frequencies?

- Next step: Populate with many more datasets (Planck HFI, S-PASS, C-BASS, CHI-PASS, DIRBE) to break degeneracies and constrain the full model. This is the main goal of `Cosmoglobe`!

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- "*BeyondPlanck*"
  - COMPET-4 program
  - PI: Hans Kristian Eriksen
  - Grant no.: 776282
  - Period: Mar 2018 to Nov 2020

Collaborating projects:

- "*bits2cosmology*"
  - ERC Consolidator Grant
  - PI: Hans Kristian Eriksen
  - Grant no: 772 253
  - Period: April 2018 to March 2023
- "*Cosmoglobe*"
  - ERC Consolidator Grant
  - PI: Ingunn Wehus
  - Grant no: 819 478
  - Period: June 2019 to May 2024

European Commission

Questions?

Beyond PLANCK

Commander

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