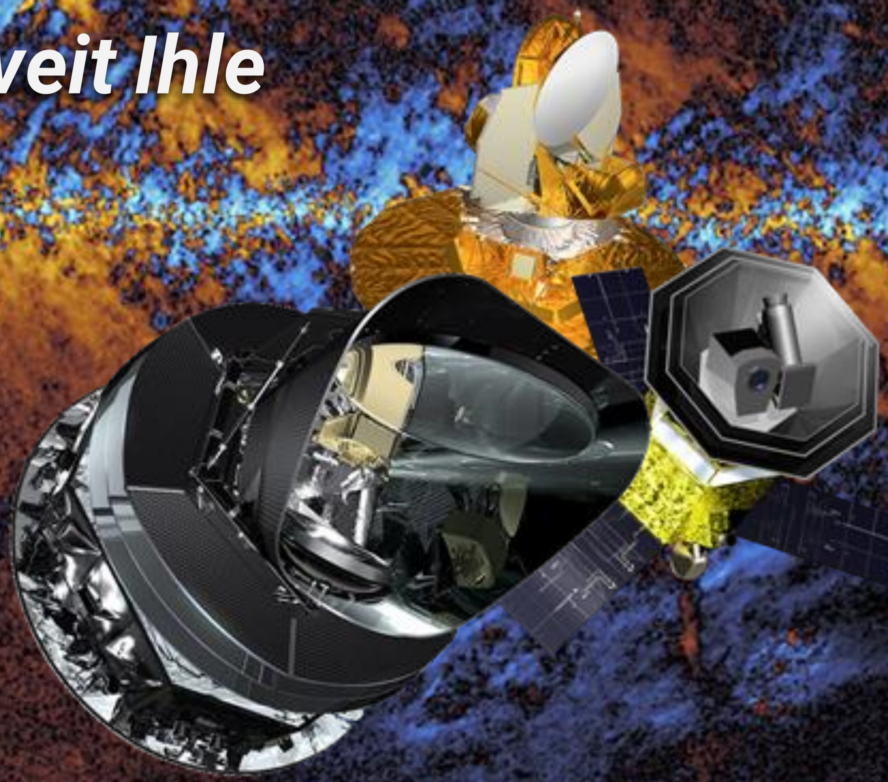
# Noise characterization and modelling

*Håvard Tveit Ihle*



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# The BeyondPlanck Gibbs sampler

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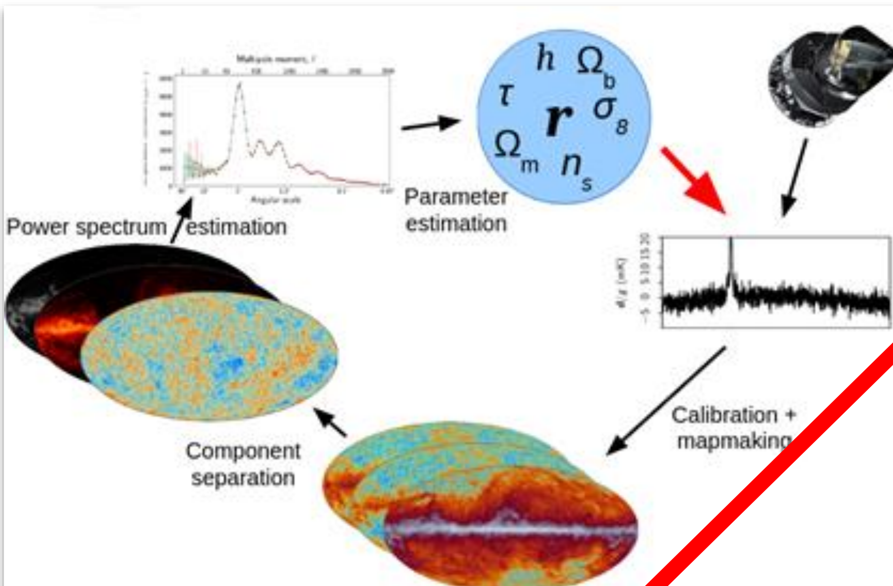
## Correlated noise, $n^{\mathrm{corr}}$

**What we want to do:**




**How we actually do it:**

$$
\begin{aligned}
\boldsymbol{g} &\leftarrow P(\boldsymbol{g} \mid \boldsymbol{d}, \xi_n, \Delta_{\mathrm{bp}}, \boldsymbol{a}, \beta, C_\ell) \\
\boldsymbol{n}_{\mathrm{corr}} &\leftarrow P(\boldsymbol{n}_{\mathrm{corr}} \mid \boldsymbol{d}, \boldsymbol{g}, \xi_n, \Delta_{\mathrm{bp}}, \boldsymbol{a}, \beta, C_\ell) \\
\xi_n &\leftarrow P(\xi_n \mid \boldsymbol{d}, \boldsymbol{g}, \boldsymbol{n}_{\mathrm{corr}}, \Delta_{\mathrm{bp}}, \boldsymbol{a}, \beta, C_\ell) \\
\Delta_{\mathrm{bp}} &\leftarrow P(\Delta_{\mathrm{bp}} \mid \boldsymbol{d}, \boldsymbol{g}, \boldsymbol{n}_{\mathrm{corr}}, \xi_n, \boldsymbol{a}, \beta, C_\ell) \\
\beta &\leftarrow P(\beta \mid \boldsymbol{d}, \boldsymbol{g}, \boldsymbol{n}_{\mathrm{corr}}, \xi_n, \Delta_{\mathrm{bp}}, C_\ell) \\
\boldsymbol{a} &\leftarrow P(\boldsymbol{a} \mid \boldsymbol{d}, \boldsymbol{g}, \boldsymbol{n}_{\mathrm{corr}}, \xi_n, \Delta_{\mathrm{bp}}, \beta, C_\ell) \\
C_\ell &\leftarrow P(C_\ell \mid \boldsymbol{d}, \boldsymbol{g}, \boldsymbol{n}_{\mathrm{corr}}, \xi_n, \Delta_{\mathrm{bp}}, \boldsymbol{a}, \beta)
\end{aligned}
$$

## Noise parameters, $\xi_n = \{\sigma_0, f_{\mathrm{knee}}, \alpha\}$

### BeyondPlanck VI. Noise characterization and modelling


H. T. Ihle[11]*, M. Bersanelli[4,9,10], C. Franceschet[4,10], E. Gjerløw[11], K. J. Andersen[11], R. Aurlien[11], R. Banerji[11], S. Bertocco[8], M. Brilenkov[11], M. Carbone[14], L. P. L. Colombo[4], H. K. Eriksen[11], M. K. Foss[11], U. Fuskeland[11], S. Galeotta[8], M. Galloway[11], S. Gerakakis[14], B. Hensley[2], D. Herman[11], M. Iacobellis[14], M. Ieronymaki[14], J. B. Jewell[12], A. Karakci[11], E. Keihänen[3,7], R. Keskitalo[1], G. Maggio[8], D. Maino[4,9,10], M. Maris[8], A. Mennella[4,9,10], S. Paradiso[4,9], B. Partridge[6], M. Reinecke[13], A.-S. Suur-Uski[3,7], T. L. Svalheim[11], D. Tavagnacco[8,5], H. Thommesen[11], D. J. Watts[11], I. K. Wehus[11], and A. Zacchei[8]



arXiv:2011.06650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# Data model

$$d_t = g s_t^{\mathrm{tot}} + n_t^{\mathrm{corr}} + n_t^{\mathrm{wn}}$$

Signal subtracted data:

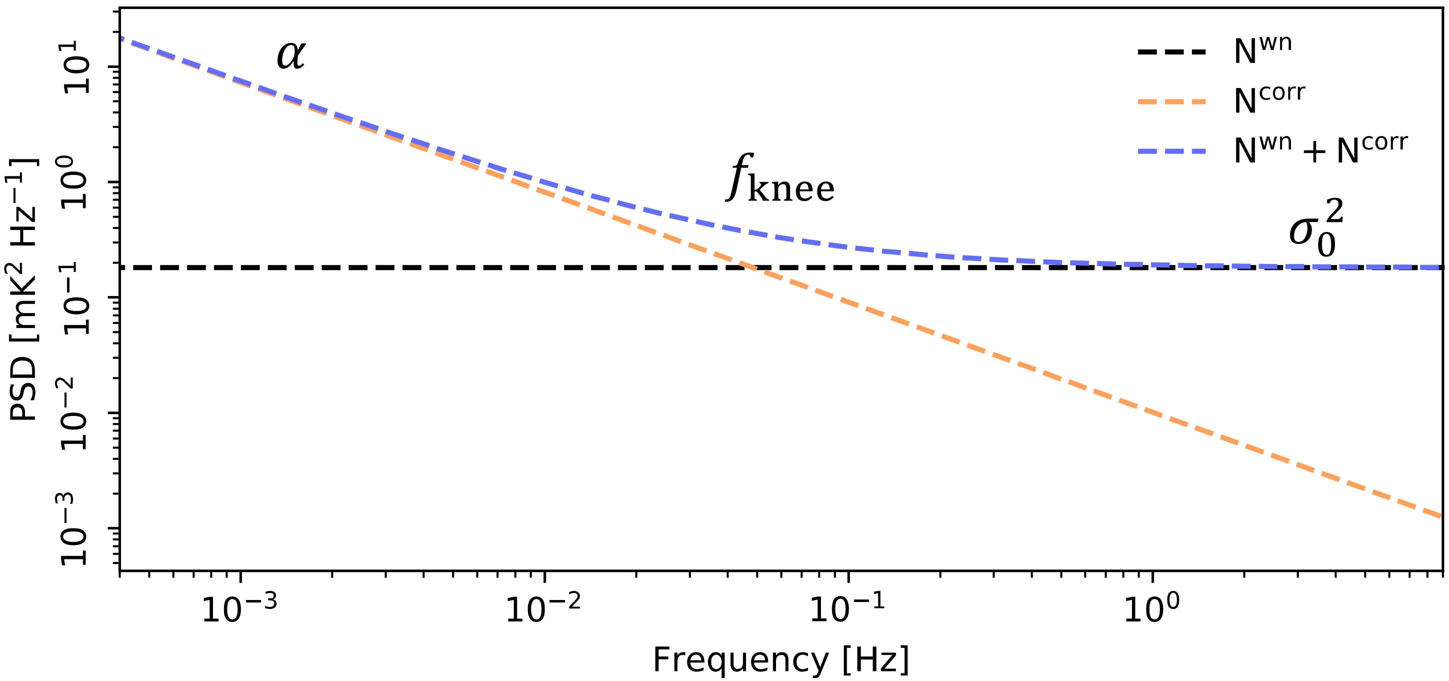
$$d'_t \equiv d_t - g s_t^{\mathrm{tot}} = n_t^{\mathrm{corr}} + n_t^{\mathrm{wn}}$$

Covariance matrices:

$$\langle n_{t'}^{\mathrm{w}} (n_t^{\mathrm{w}})^T \rangle = \mathrm{N^{wn}}_{tt'} = \sigma_0^2$$

$$\langle n_{t'}^{\mathrm{corr}} (n_t^{\mathrm{corr}})^T \rangle = \mathrm{N^{corr}}_{tt'} \overset{\mathrm{Fourier}}{\overbrace{\Rightarrow}} \sigma_0^2 \left( \frac{f}{f_{\mathrm{knee}}} \right)^{\alpha}$$

# $1/f$ – model




$$\langle n^{\mathrm{w}}_{t'} (n^{\mathrm{w}}_t)^T \rangle = \mathrm{N^{wn}}_{tt'} = \sigma_0^2$$

$$\langle n^{\mathrm{corr}}_{t'} (n^{\mathrm{corr}}_t)^T \rangle = \mathrm{N^{corr}}_{tt'} \overset{\text{Fourier}}{\overbrace{\Rightarrow}} \sigma_0^2 \left( \frac{f}{f_{\mathrm{knee}}} \right)^{\alpha}$$

# Sampling the correlated noise

$$(\mathrm{N}_{\mathrm{wn}}^{-1} + \mathrm{N}_{\mathrm{corr}}^{-1})n^{\mathrm{corr}} = \mathrm{N}_{\mathrm{wn}}^{-1}d' + \mathrm{N}_{\mathrm{wn}}^{-1/2}w_1 + \mathrm{N}_{\mathrm{corr}}^{-1/2}w_2$$

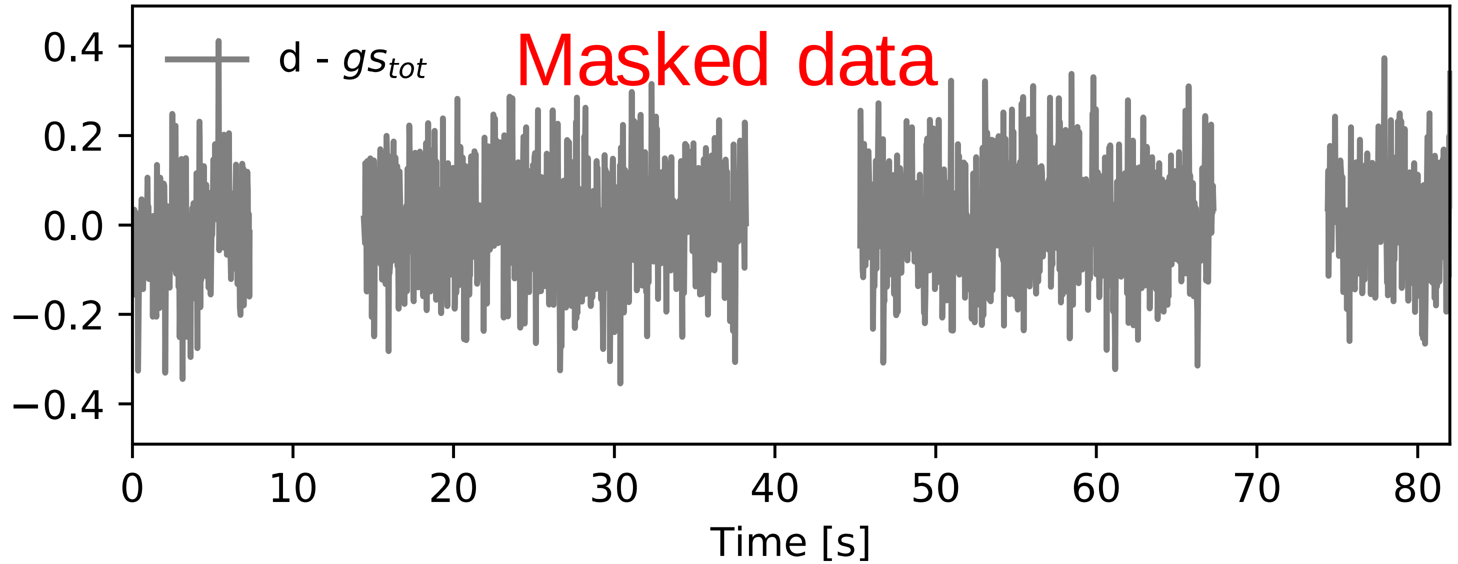
$$w_i \sim \mathrm{N}(0, \sigma)$$

Maximum Likelihood

Sampling terms

Solve in Fourier space:

$$n_f^{\mathrm{corr}} = \frac{d_f' + \mathrm{N}_{\mathrm{wn}}^{1/2}w_1 + \mathrm{N}_{\mathrm{wn}}\mathrm{N}_{\mathrm{corr}}^{-1/2}w_2}{1 + \mathrm{N}_{\mathrm{wn}}/\mathrm{N}_{\mathrm{corr}}}$$



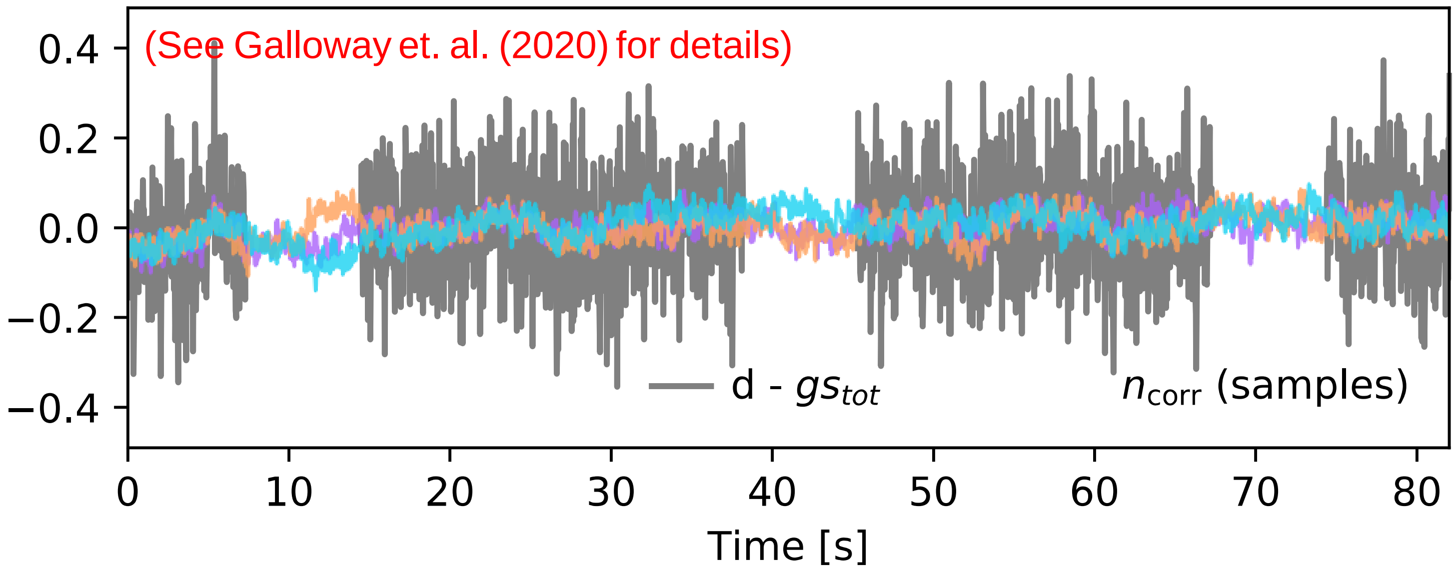
# Sampling the correlated noise

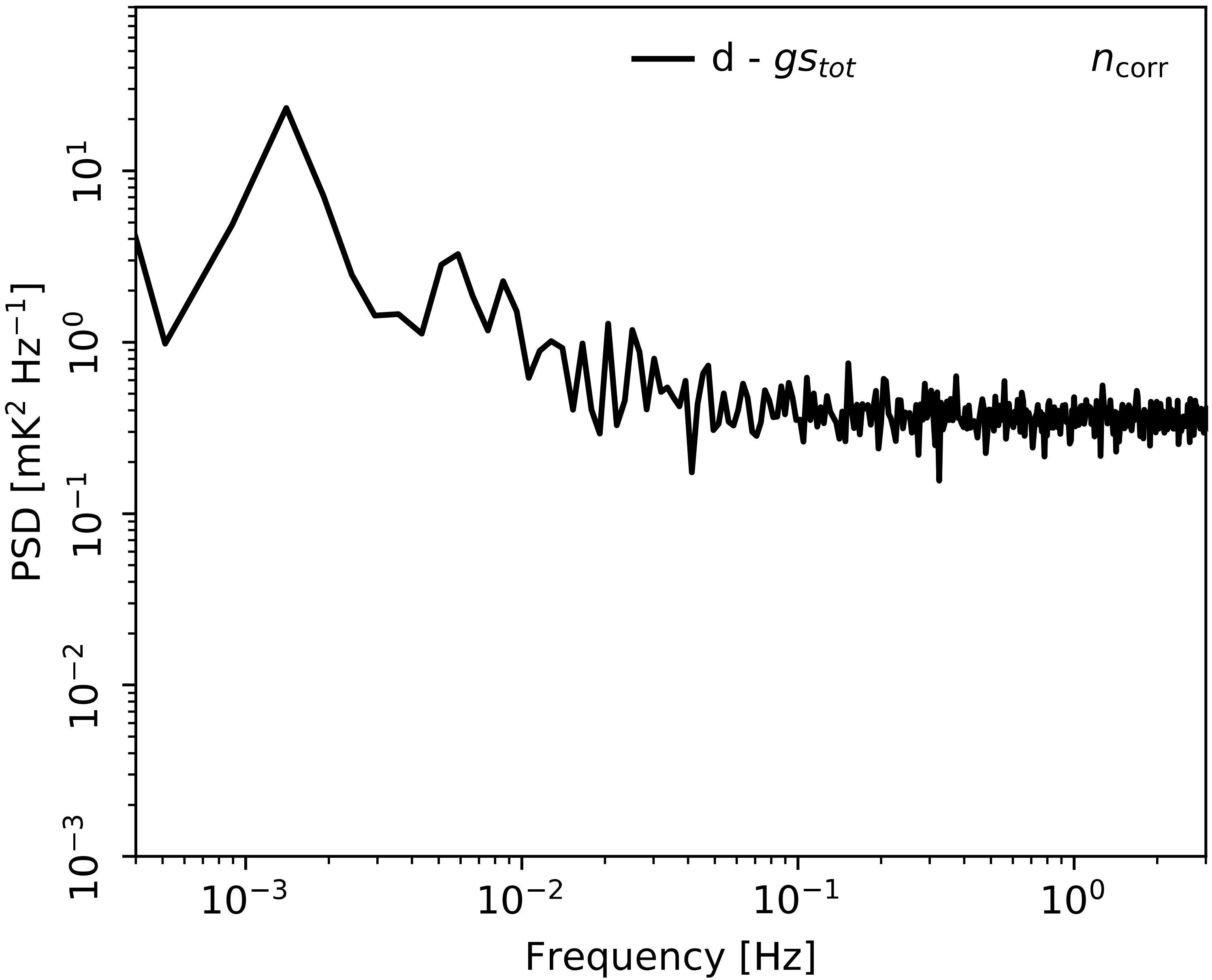
$$\underbrace{(\mathrm{N}_{\mathrm{wn}}^{-1} + \mathrm{N}_{\mathrm{corr}}^{-1})}_{A}\underbrace{n^{\mathrm{corr}}}_{x} = \underbrace{\mathrm{N}_{\mathrm{wn}}^{-1} d' + \mathrm{N}_{\mathrm{wn}}^{-1/2} w_1 + \mathrm{N}_{\mathrm{corr}}^{-1/2} w_2}_{b}$$

$$A \quad x \quad = \quad b$$

Solve by iterative conjugate gradient (CG) method:

Costly, but doable, in terms of CPU-time! (See Keihänen et. al. (2020) for details)





d - $gs_{tot}$
$n_{corr}$
PSD [mK$^2$ Hz$^{-1}$]
$10^{1}$
$10^{0}$
$10^{-1}$
$10^{-2}$
$10^{-3}$
$10^{-3}$
$10^{-2}$
$10^{-1}$
$10^{0}$
Frequency [Hz]

# Noise power spectrum sampling

Need to sample noise PSD parameters ( $\xi_n = \{\sigma_{0,} f_{\text{knee}}, \alpha\}$ )

(Sampled independently for each ~ 1 hour pointing period (PID))

We use the following trick to fix $\sigma_0$ (so $\sigma_0$ is not sampled!)

$$\sigma_0^2 \equiv \frac{\mathrm{Var}(r_i - r_{i-1})}{2}, \quad r_i \equiv d' - n^{\text{corr}}$$

For the correlated noise parameters we sample from the likelihood

$$P(\xi^n \setminus \sigma_0 | n^{\text{corr}}) \propto \frac{1}{\sqrt{|\mathrm{N}_{\text{corr}}|}} \exp\left(-\frac{1}{2}(n^{\text{corr}})^T \mathrm{N}_{\text{corr}}^{-1} n^{\text{corr}}\right)$$

where

$$\mathrm{N}_{\text{corr}}(\xi^n) = \langle n^{\text{corr}} (n^{\text{corr}})^T \rangle \overset{\text{Fourier}}{\Rightarrow} \sigma_0^2 \left(\frac{f}{f_{\text{knee}}}\right)^{\alpha}$$

# Priors on $f_{\mathrm{knee}}$ and $\alpha$

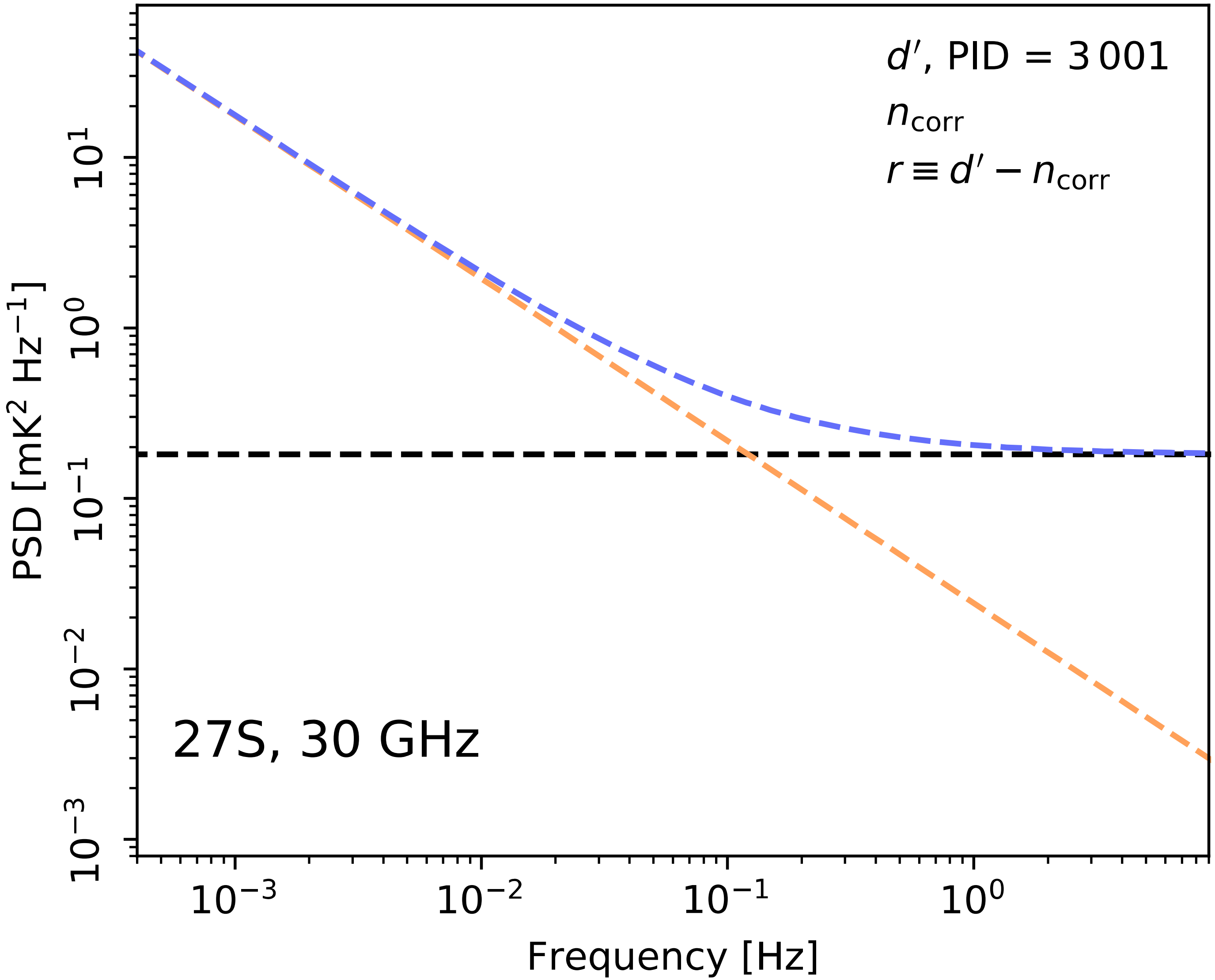
Use values for $f_{\mathrm{knee}}$ and $\alpha$ derived in the main Planck LFI DPC analysis (Planck Collaboration II. 2020)

$$-2\ln P(\alpha) = \left(\frac{\alpha - \alpha^{\mathrm{DPC}}}{\sigma_\alpha}\right)^2,$$

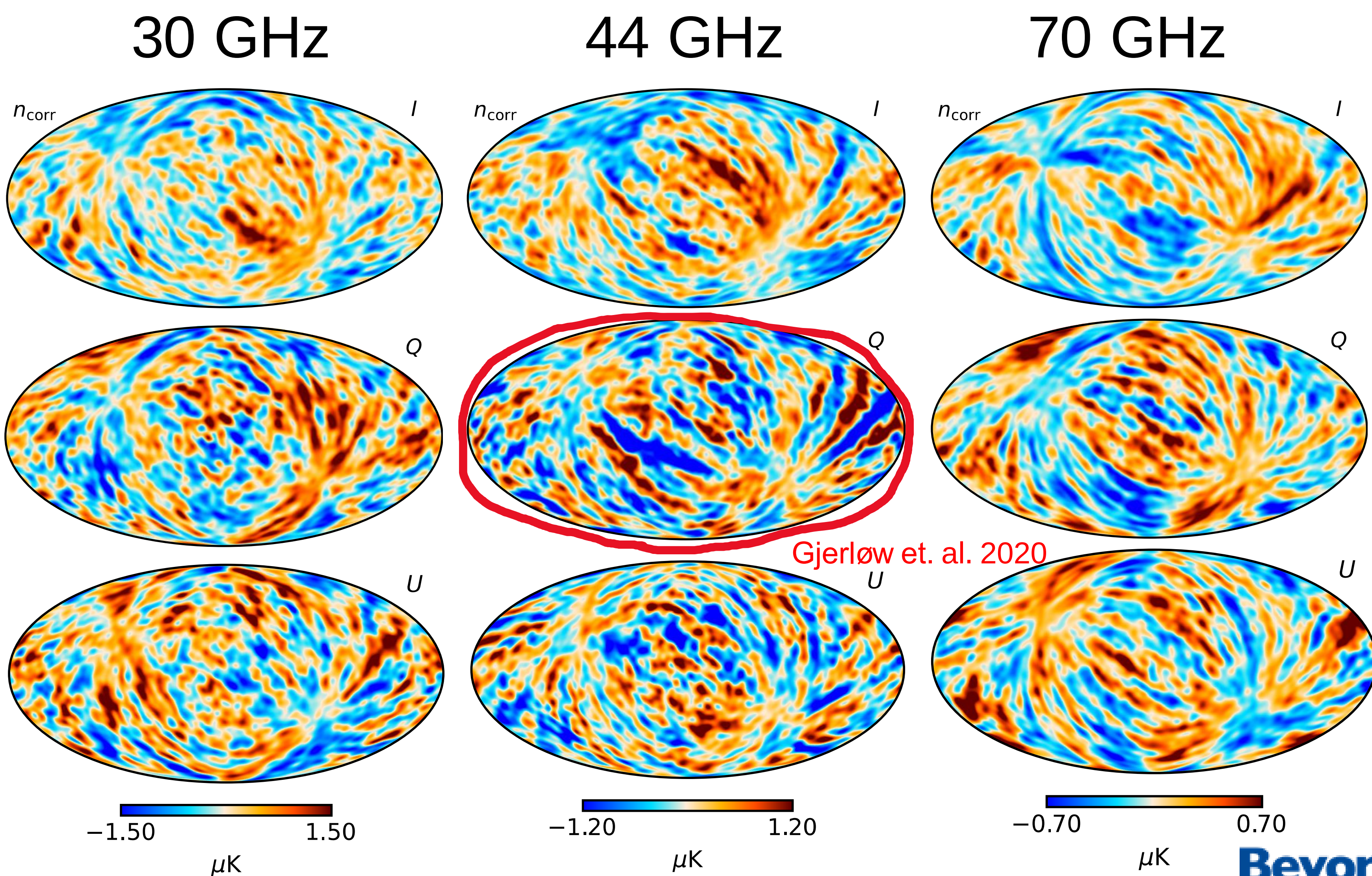
$$-2\ln P(f_{\mathrm{knee}}) = \left(\frac{\log_{10} f_{\mathrm{knee}} - \log_{10} f_{\mathrm{knee}}^{\mathrm{DPC}}}{\sigma_{f_{\mathrm{knee}}}}\right)^2 + 2\ln f_{\mathrm{knee}},$$

$\sigma_\alpha = 0.2$, $\sigma_{f_{\mathrm{knee}}} = 0.1$.

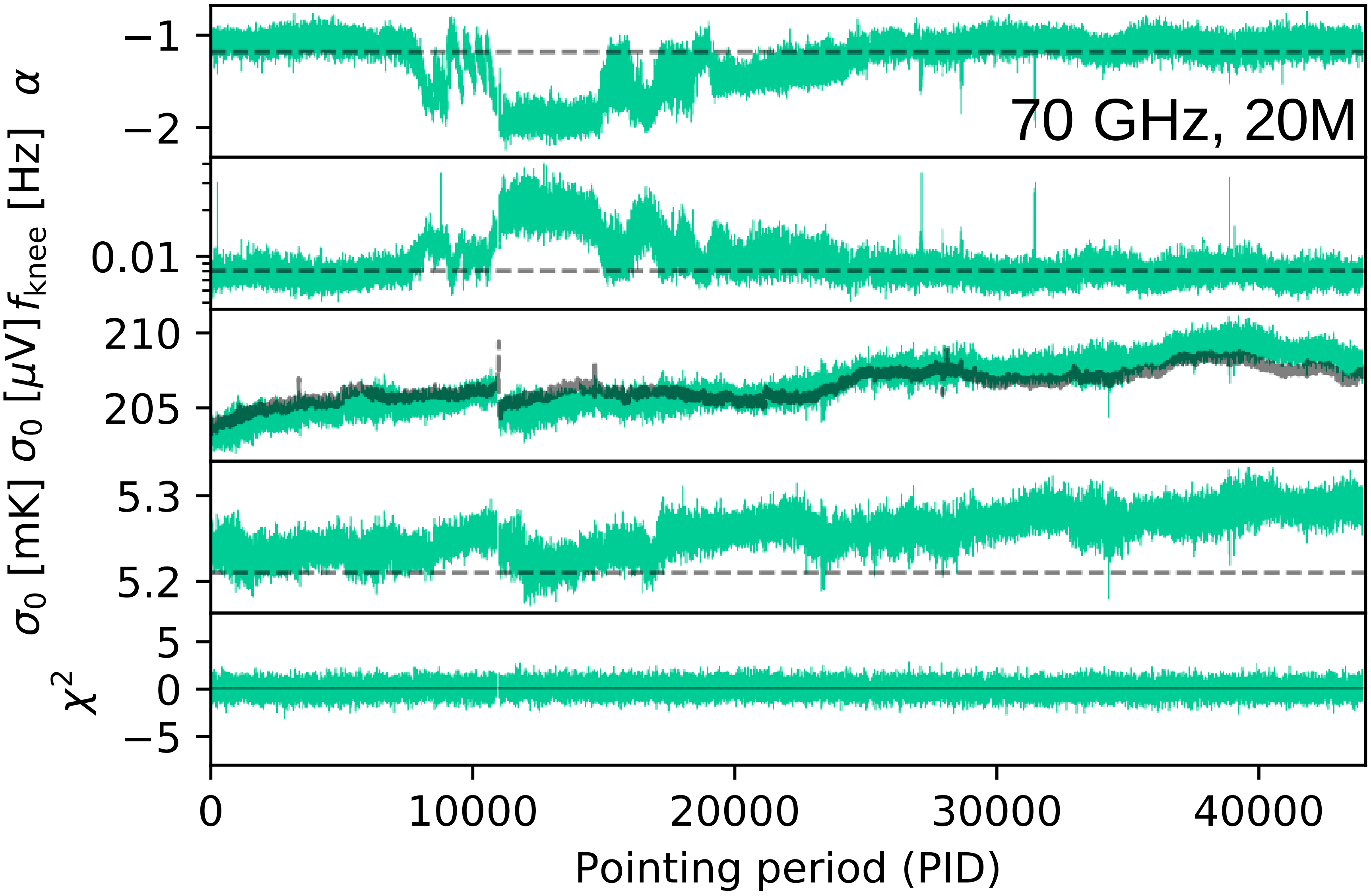
# Example of full noise model



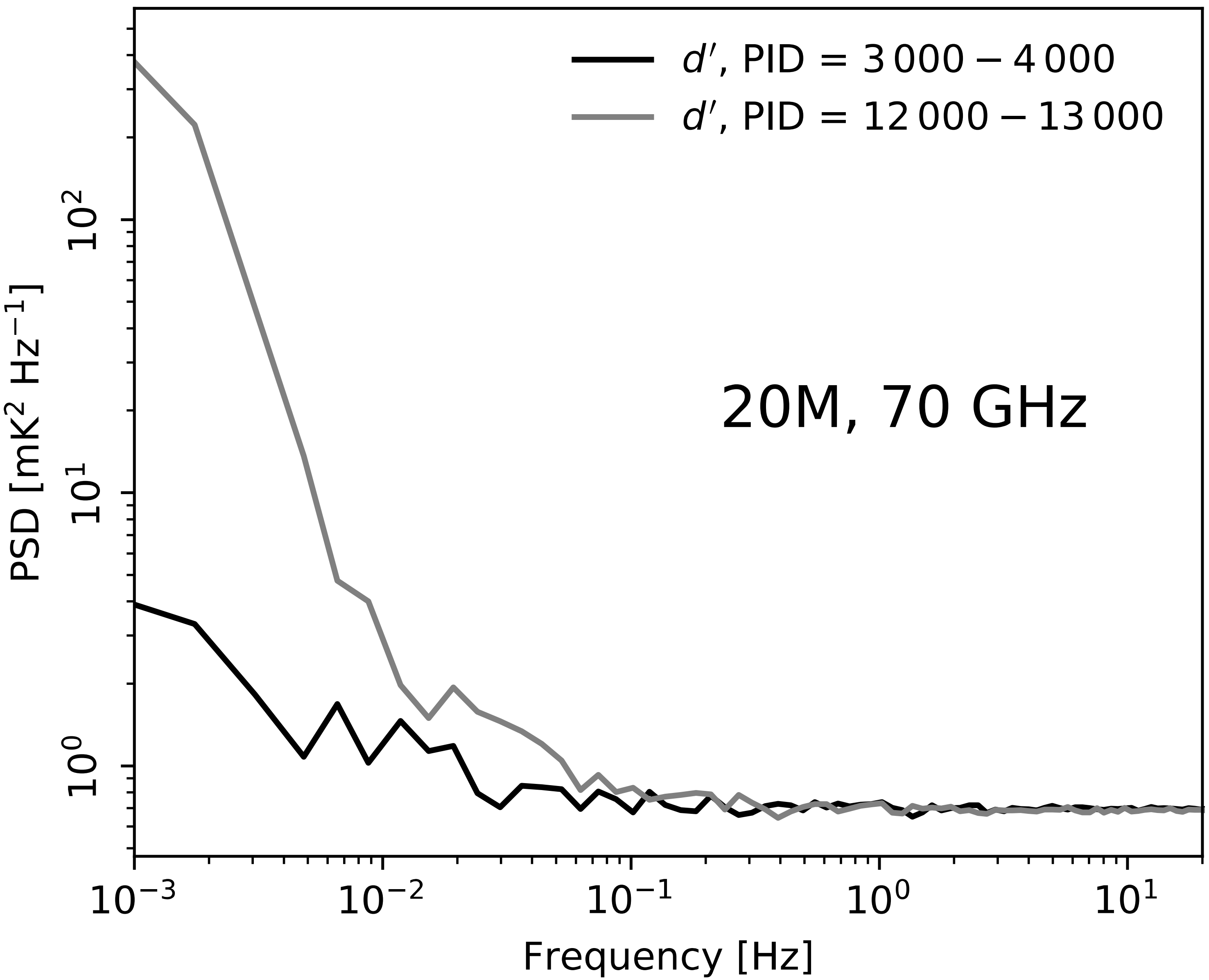
# Correlated noise maps (one Gibbs sample)



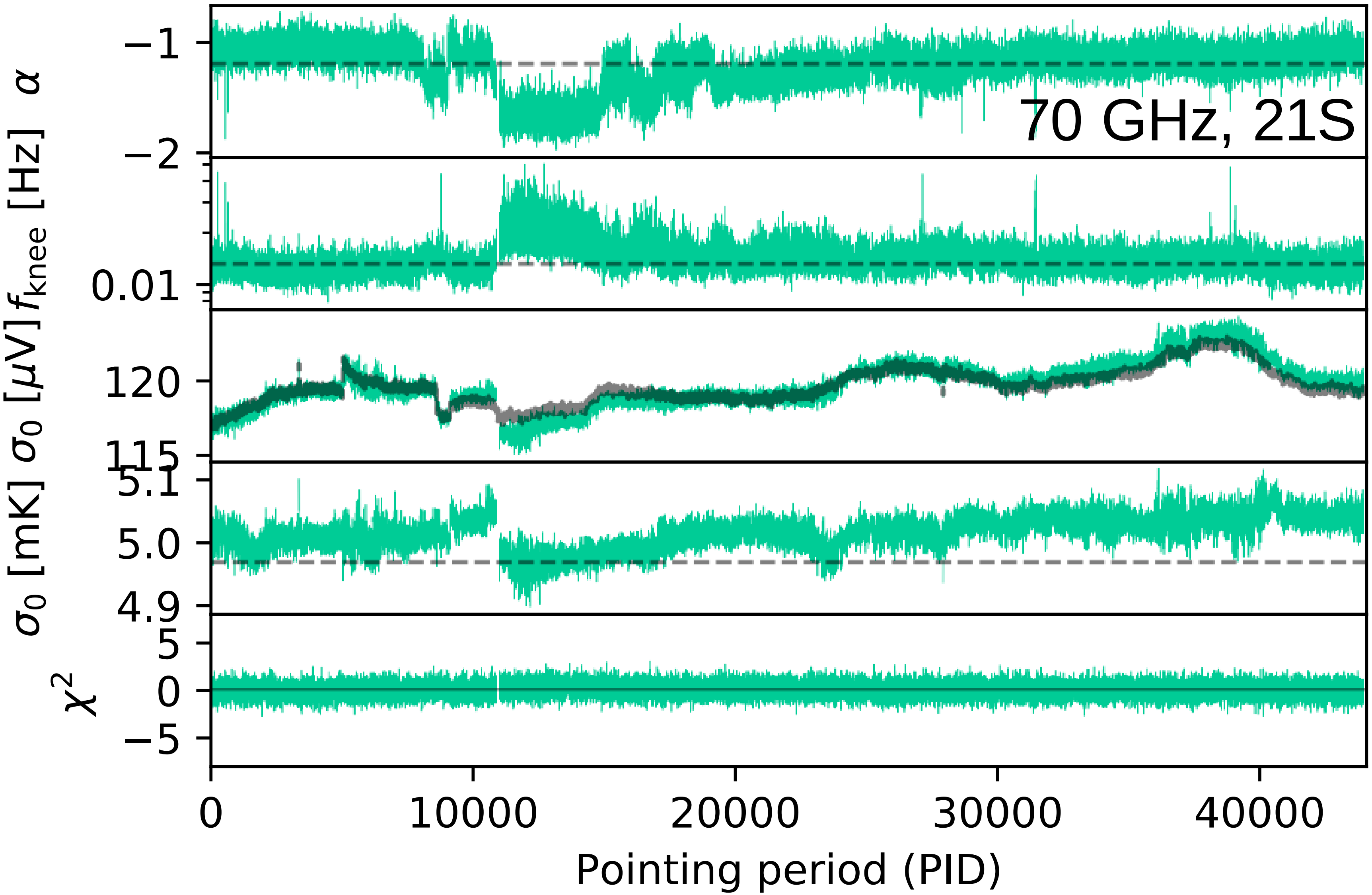
# Evolution of noise parameters



# Evolution of noise parameters



# Evolution of noise parameters



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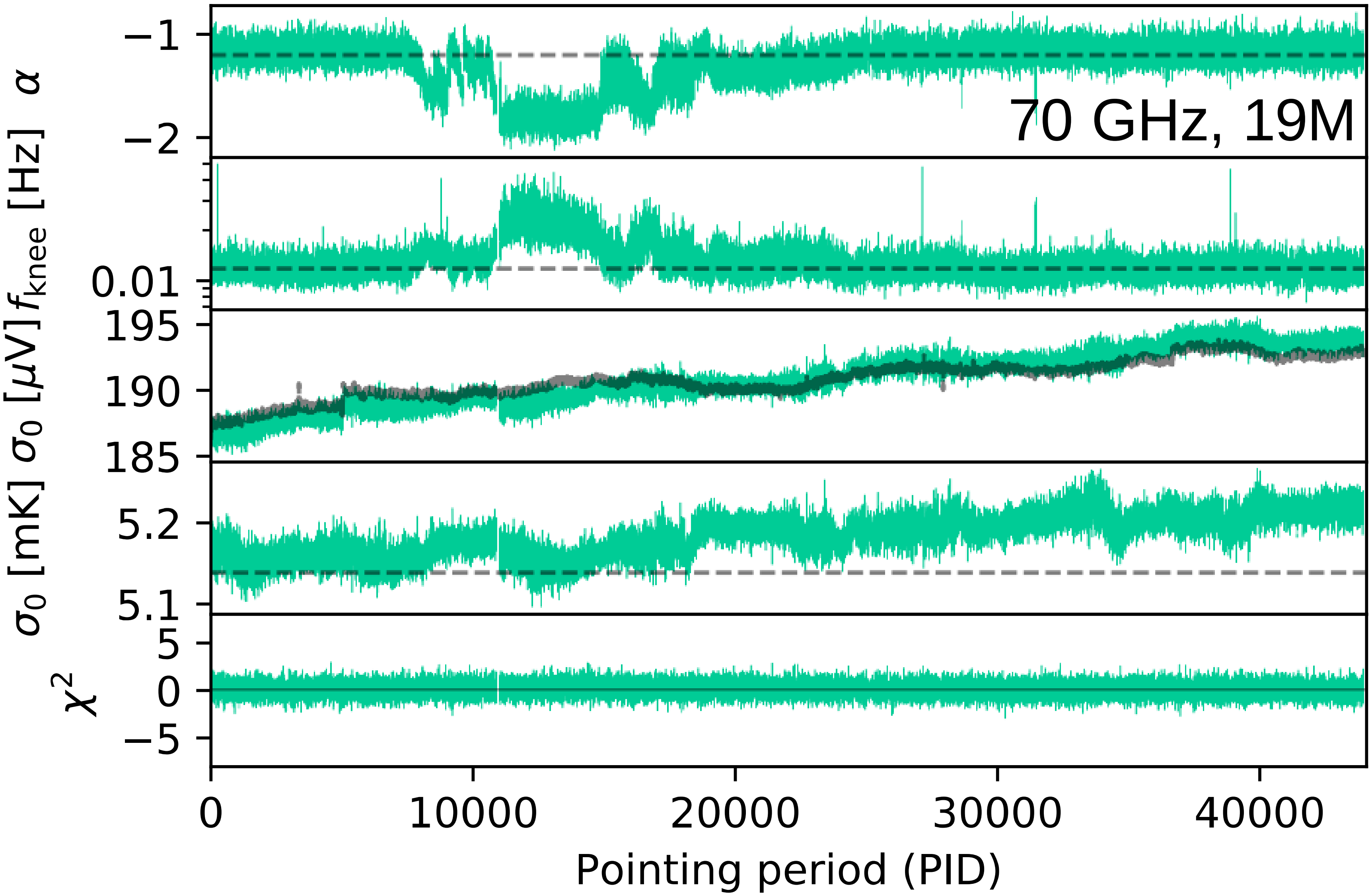
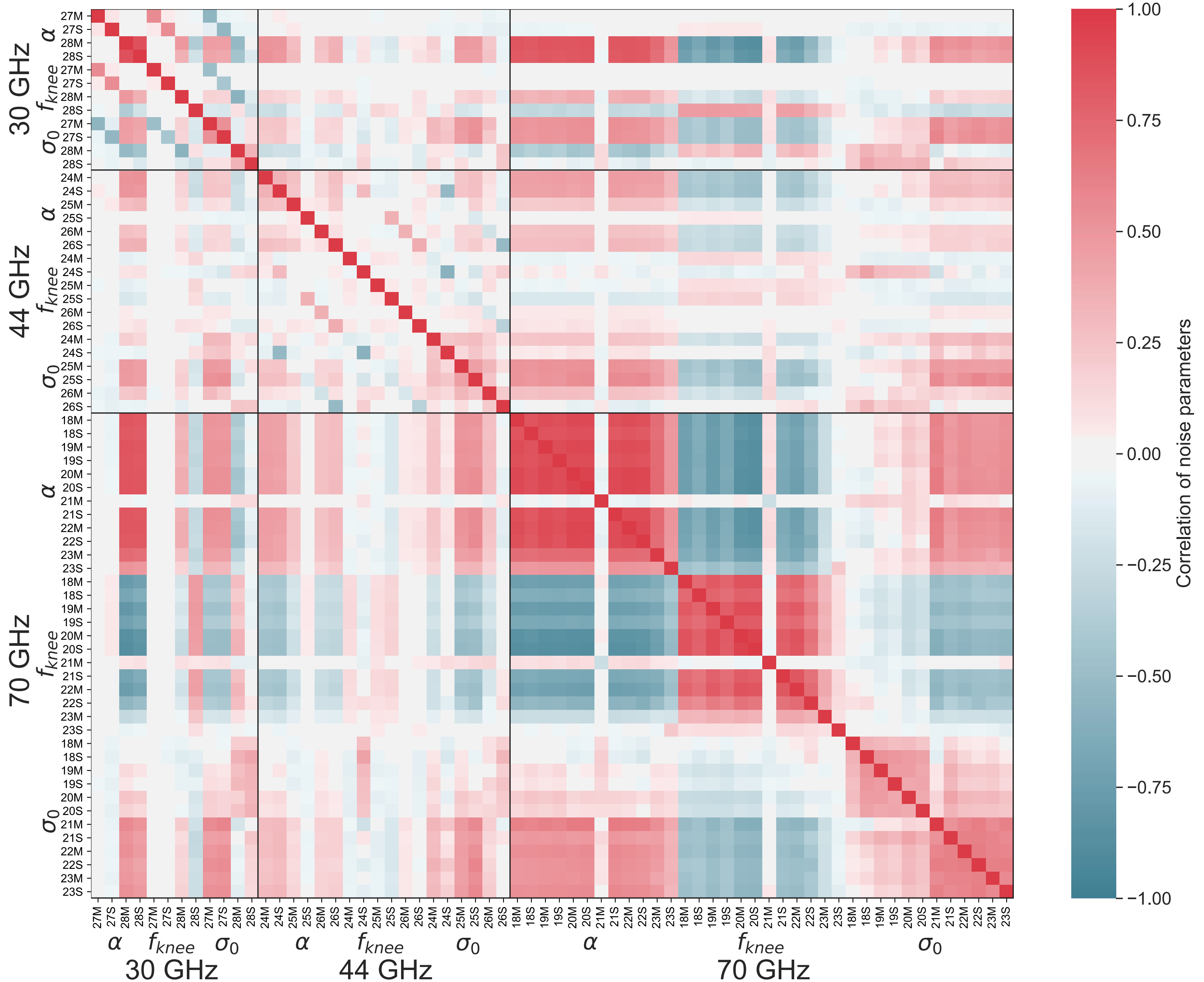
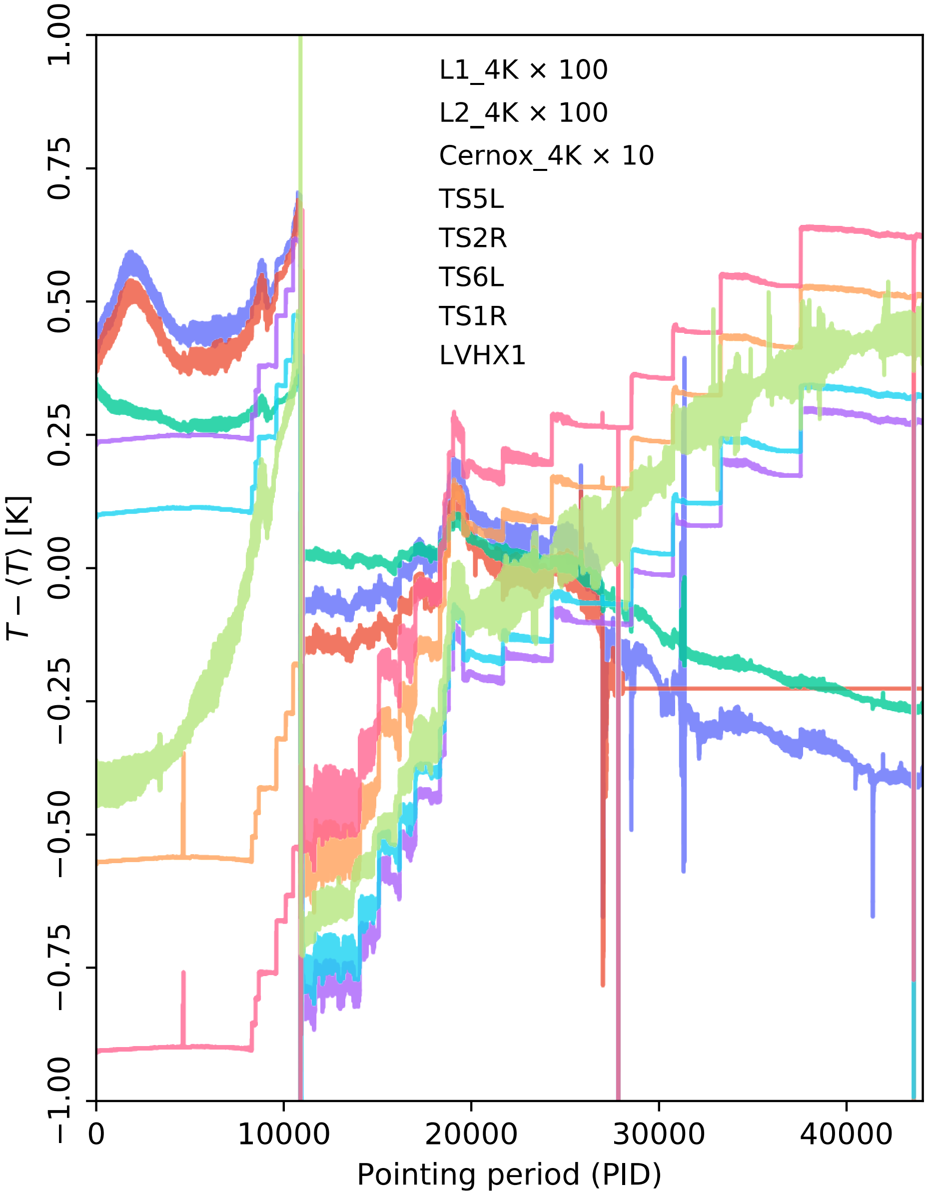


# Inter-radiometer correlations of noise parameters



# Temperature sensors on the Planck Instrument



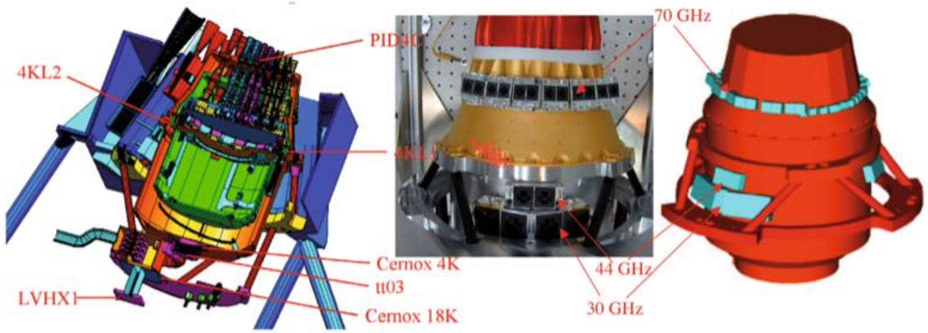
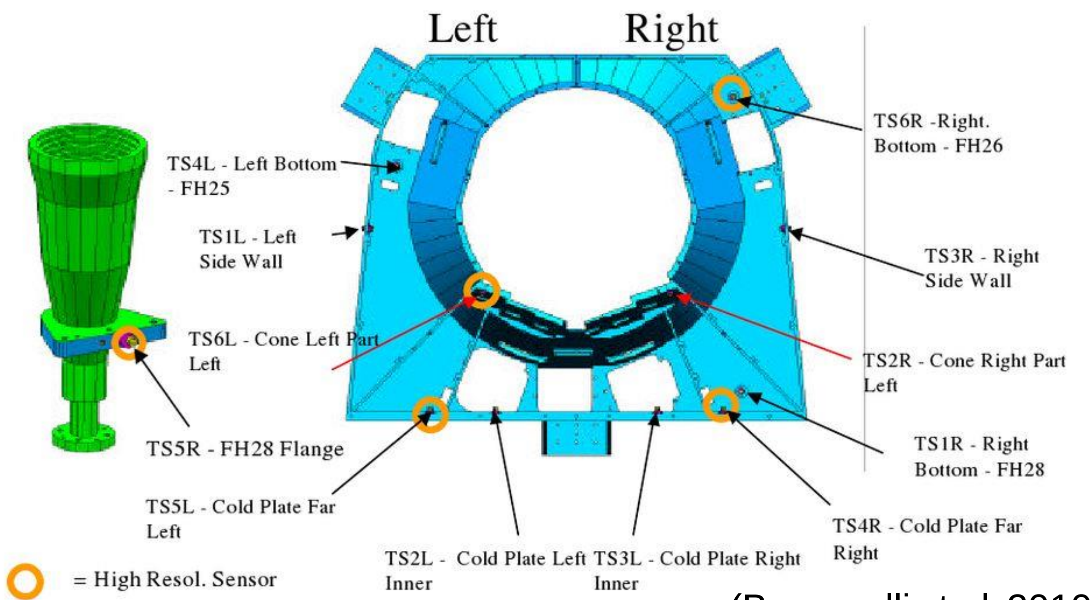




**Fig. 18.** Position of thermometers (*left*) and of the reference loads for the LFI (*centre and right*). (Lamarre et. al. 2010)

**Focal Plane Temperature Sensor Position**




(Bersanelli et.al. 2010)

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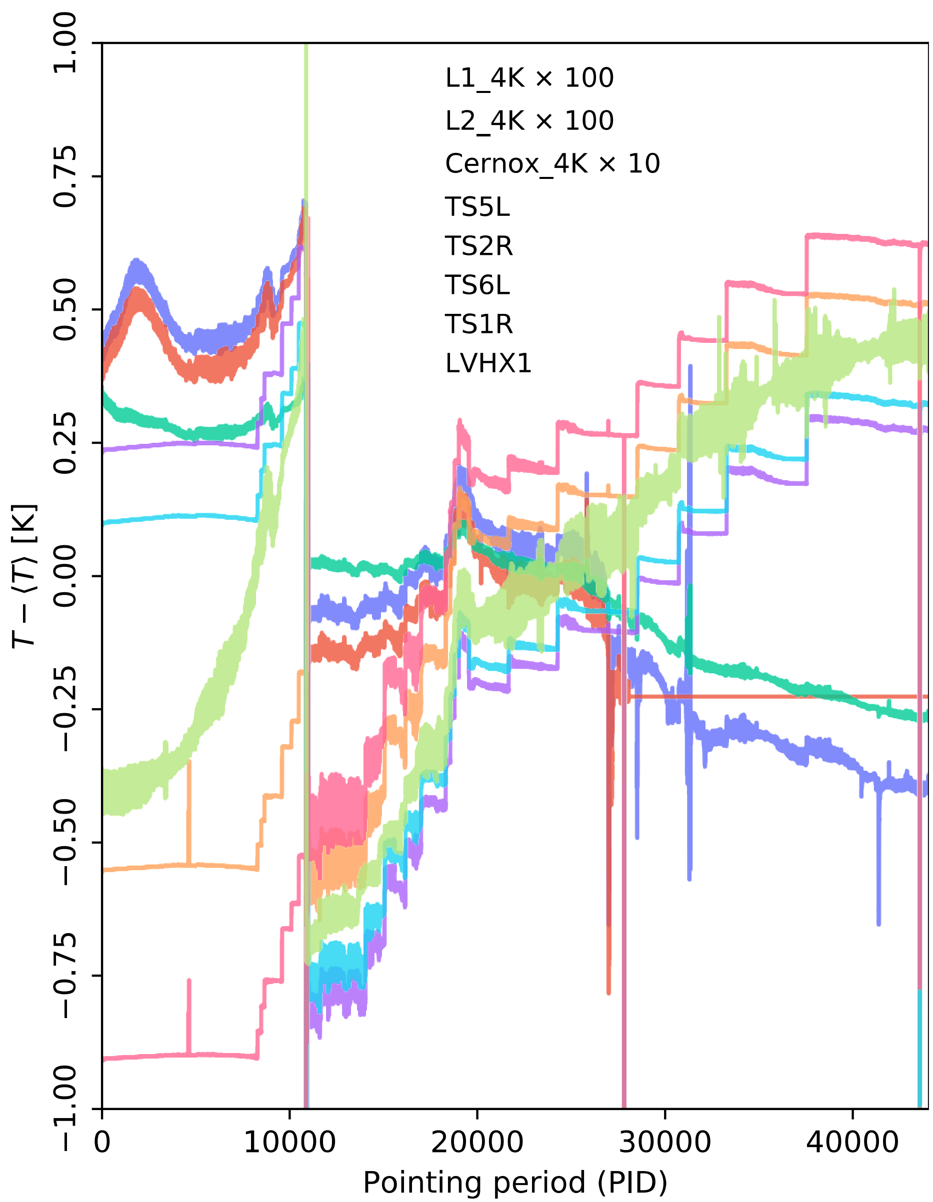
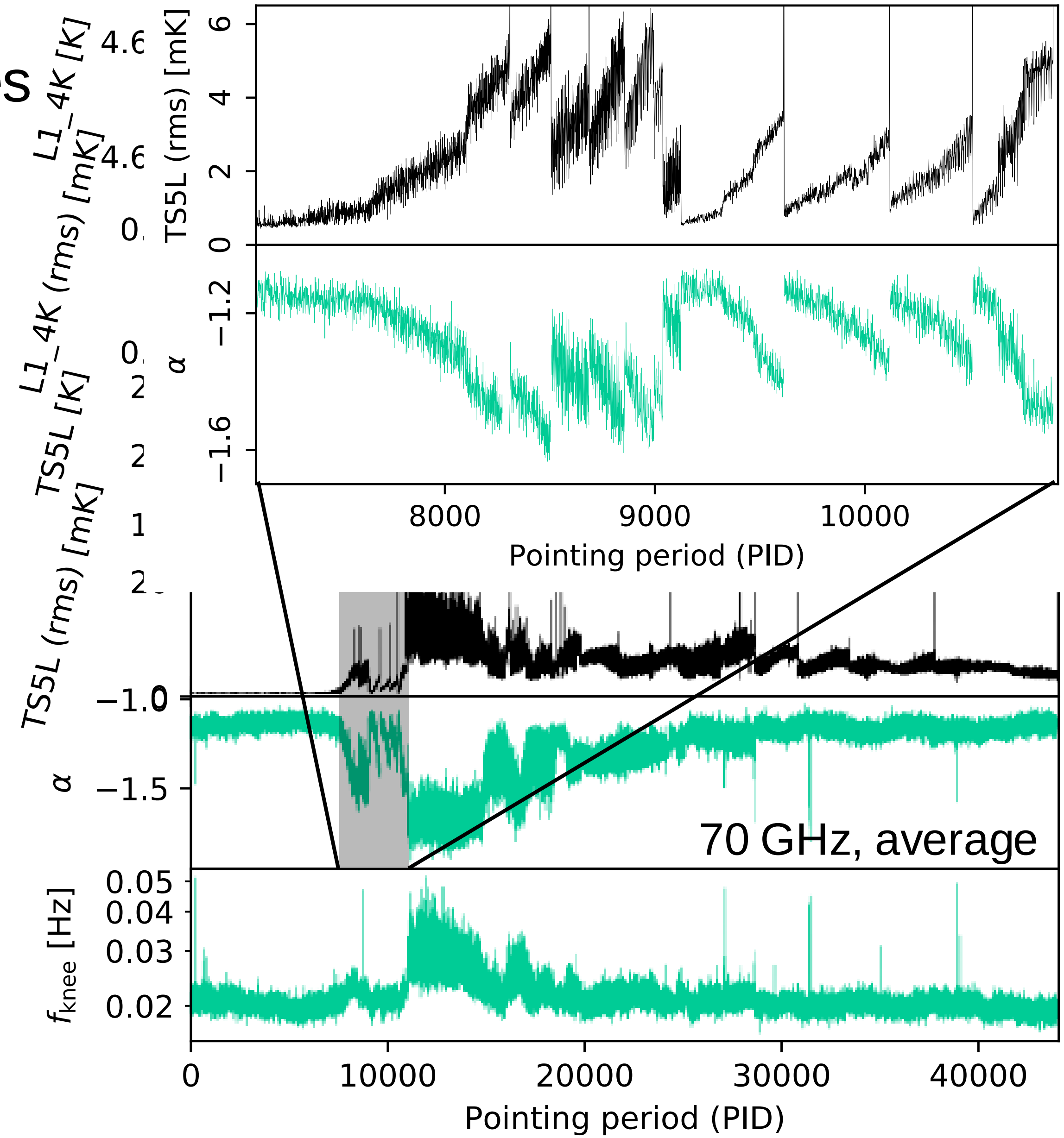
# Relating temperature sensors to noise properties

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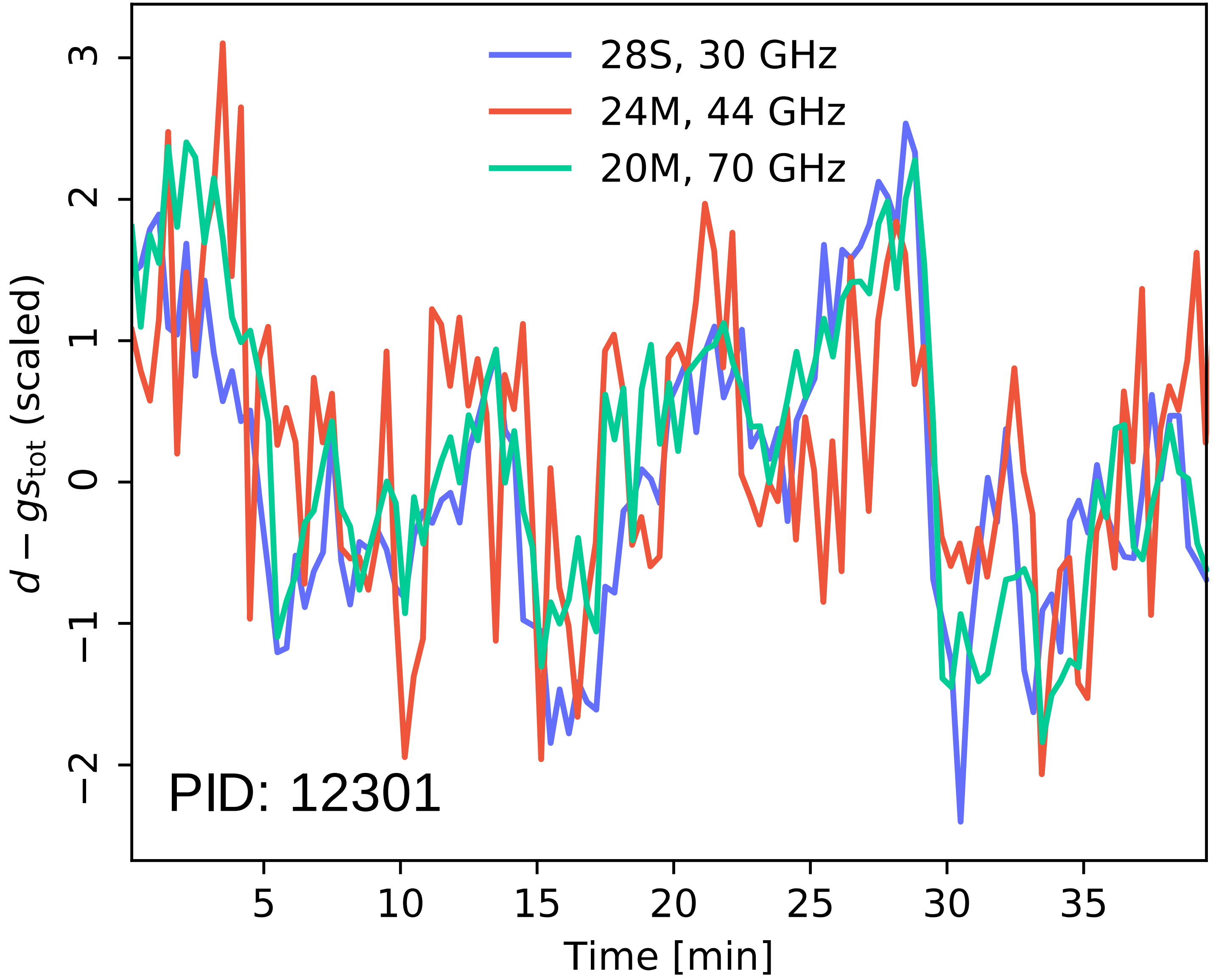
# Inter-radiometer correlation of the correlated noise?



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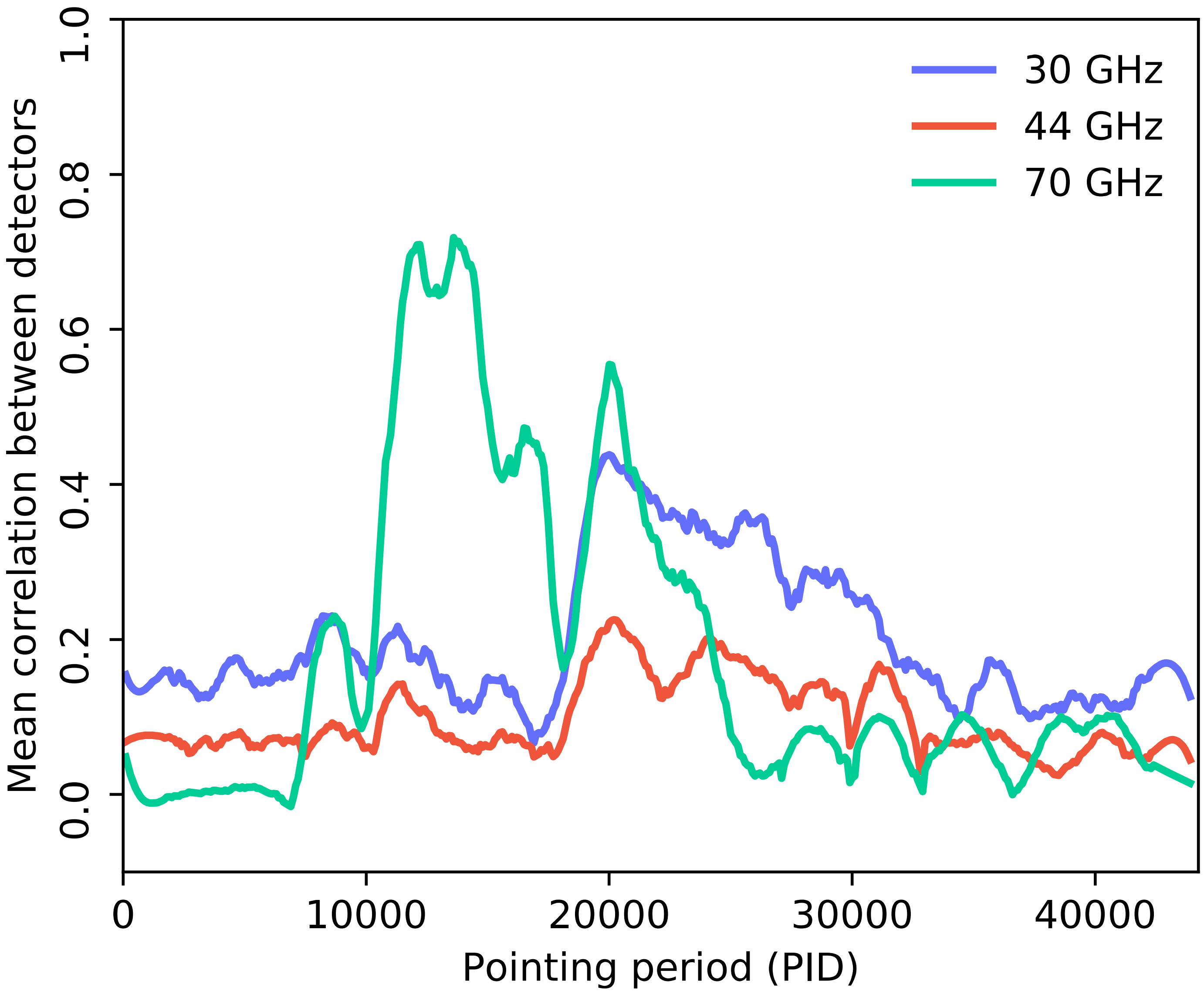

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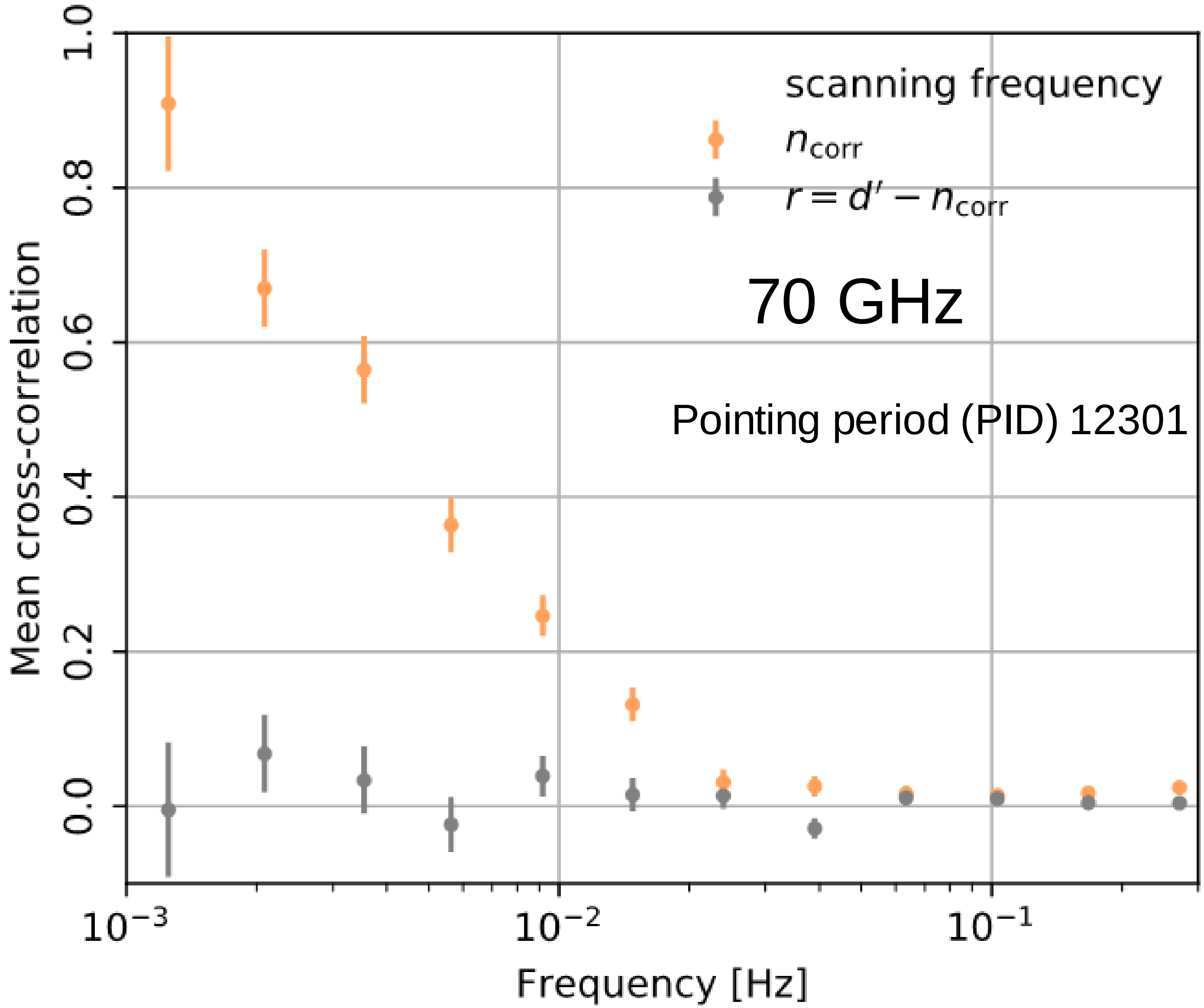
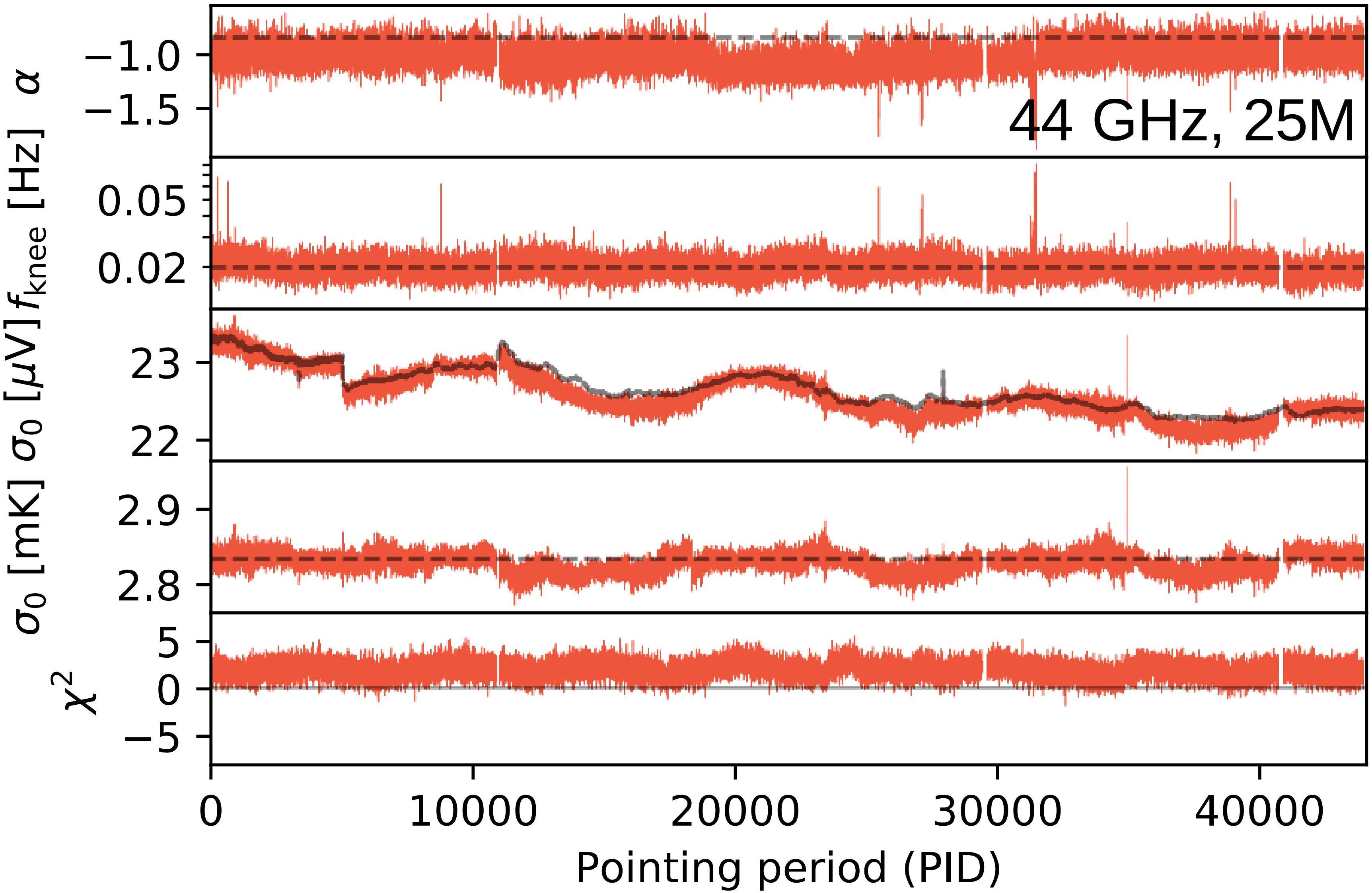
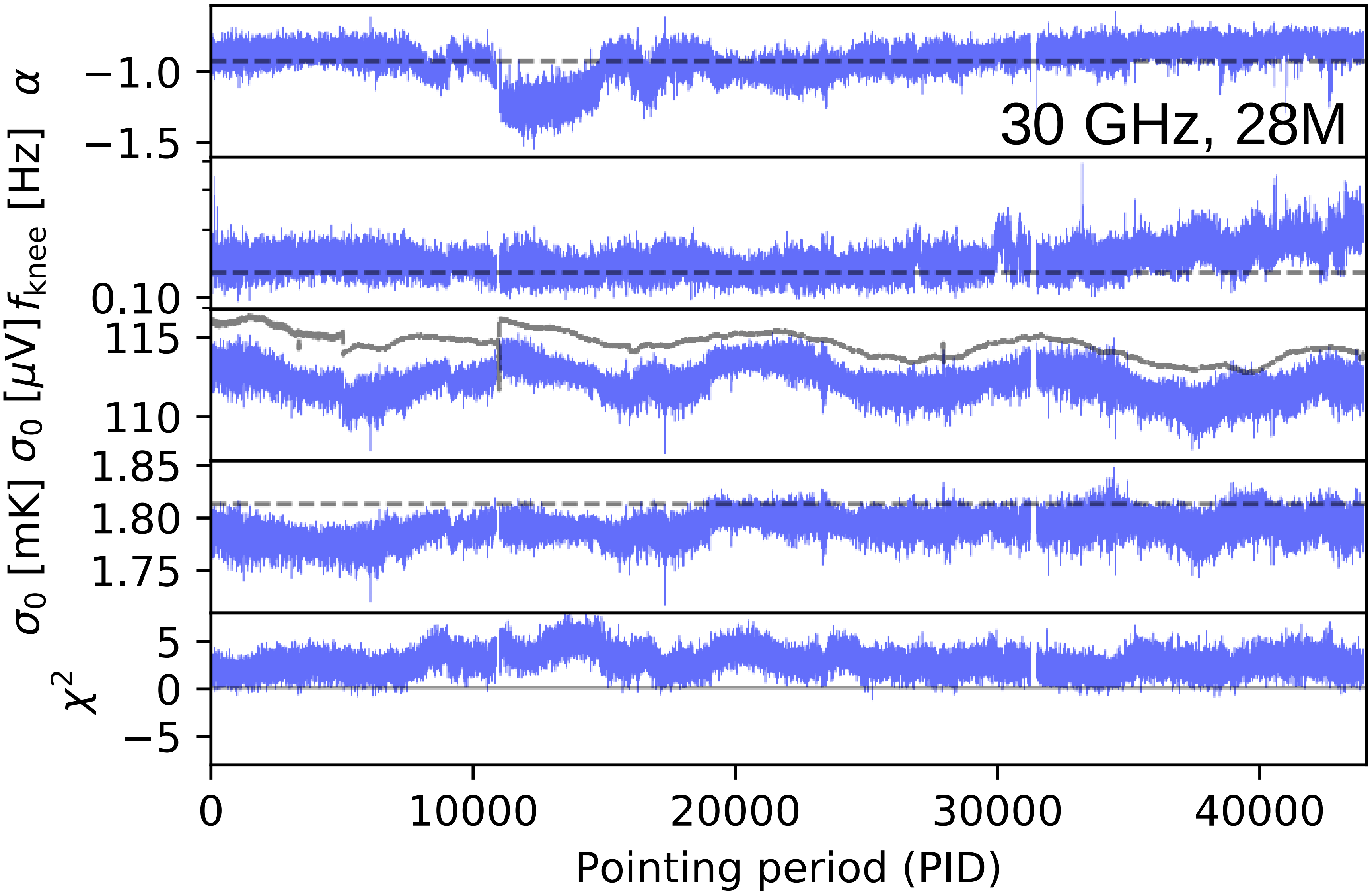


# Evolution of noise parameters



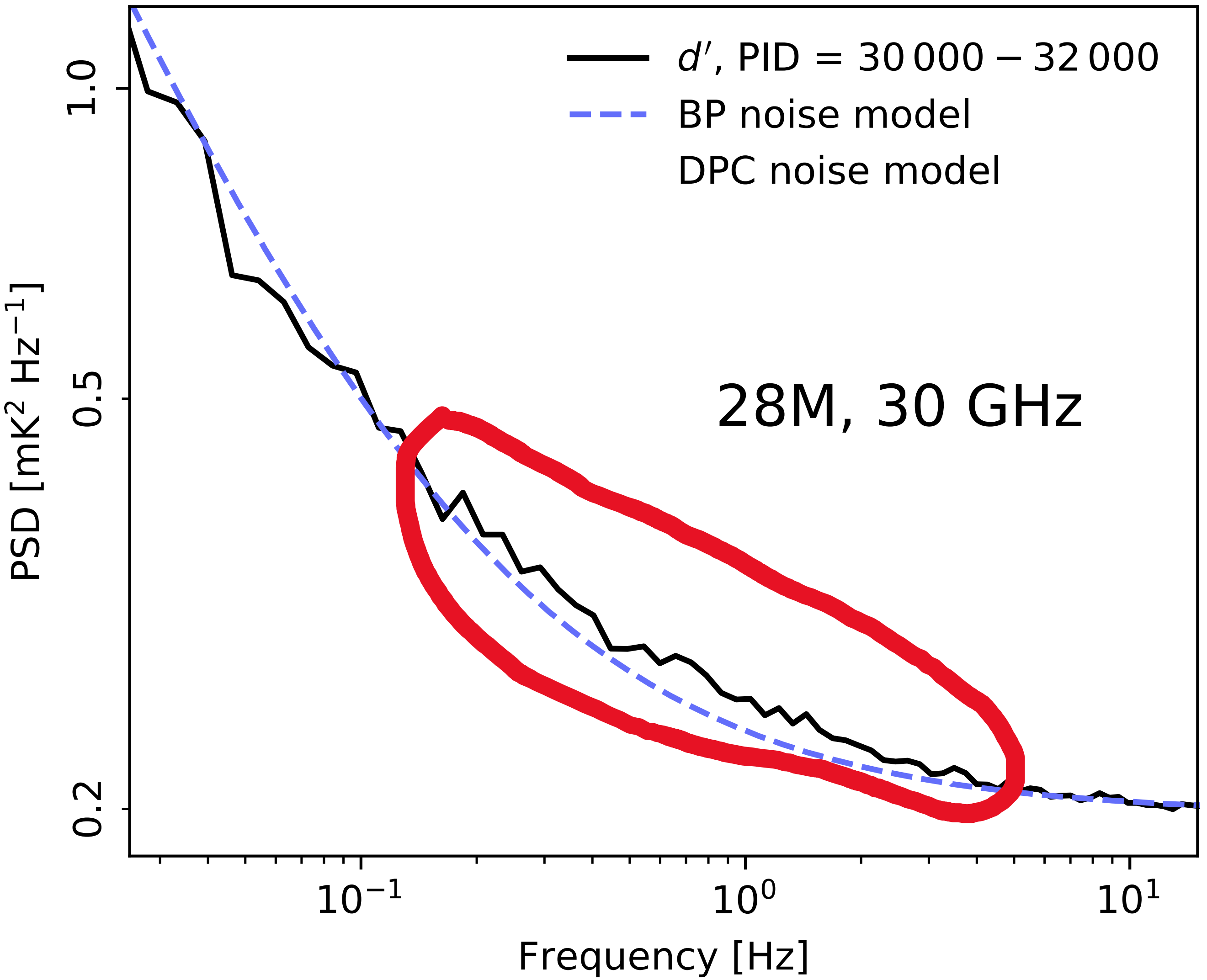
# Evolution of noise parameters



# Limits of the $1/f$ model?

$$P(f) = \sigma_0^2 \left(1 + \left(\frac{f}{f_{\mathrm{knee}}}\right)^{\alpha}\right)$$

Temporal frequencies 0.1 - 5 Hz, corresponds roughly to angular scales of 1-60 degrees. Our main science range!



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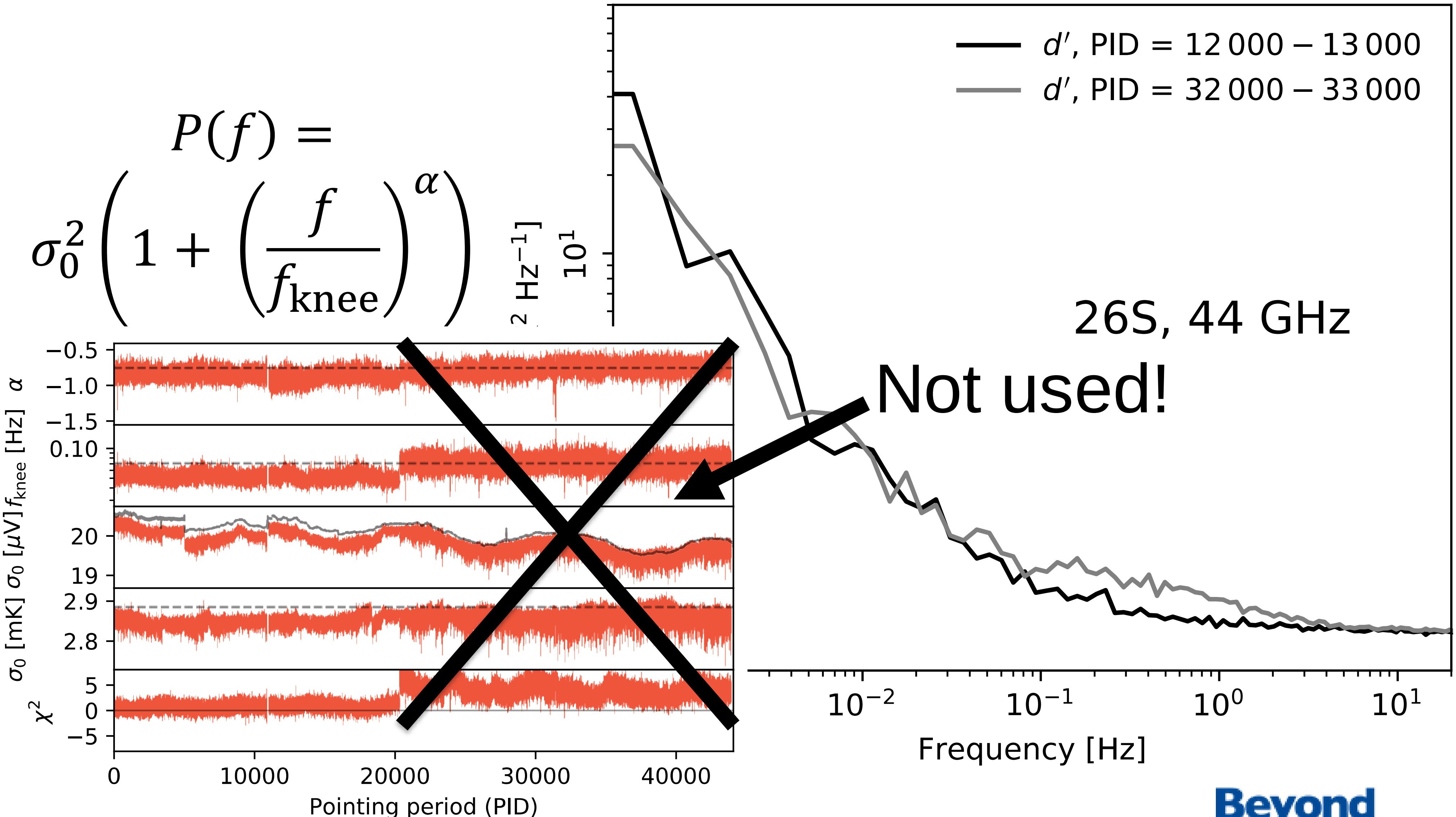
$$P(f) = \sigma_0^2\left(1 + \left(\frac{f}{f_{\mathrm{knee}}}\right)^{\alpha}\right)$$



# Summary

- Gap – filling using CG is costly, but works great for correlated noise sampling
- Noise properties change significantly over time as the LFI thermal environment changes
- At times large correlations between the noise of the different radiometers (mostly on long timescales)
- Residual issues for 30 and 44 GHz. Breakdown of 1/f - model?

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- "*BeyondPlanck*"
  - COMPET-4 program
  - PI: Hans Kristian Eriksen
  - Grant no.: 776282
  - Period: Mar 2018 to Nov 2020

Collaborating projects:

- "*bits2cosmology*"
  - ERC Consolidator Grant
  - PI: Hans Kristian Eriksen
  - Grant no: 772 253
  - Period: April 2018 to March 2023
- "*Cosmoglobe*"
  - ERC Consolidator Grant
  - PI: Ingunn Wehus
  - Grant no: 819 478
  - Period: June 2019 to May 2024

# Questions?

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Commander

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